考試科目	統計學B	系所別	金融學系財務工程與金融科技組	考試時間	2月4日(四) 第三節

Short Answer Questions

Write your answers on the answer sheet. No need to provide details unless otherwise told. Each blank worths 5 points.

■ In a survey of 300 visitors, 165 responded that they prefer to go to the beach and 135 responded that they prefer to stay in their hotels. Let p denote the fraction of all visitors who prefer to go to the beach and $\Phi(\cdot)$ be the cumulative probability density function of the standard normal distribution. The approximate p-value for the test $H_0 : p = 0.5$ versus $H_1 : p > 0.5$ in terms of $\Phi(\cdot)$ is ___(1)___. If the true p is 0.6 and the significance level $\alpha = 0.05$, the power of this test in terms of $\Phi(\cdot)$ is ___(2)___.

■ Let X be the number of weeks before the prices change in a grocery store and p be the probability of updating the prices in a given week. The pricing decision is independent between any two weeks. If the distribution of X is described by

$$f(x) = \binom{x-1}{r-1} p^r (1-p)^{x-r},$$

where $x = r, r+1, \ldots$ The average duration of prices in this store is ___(3)___ if $r = 1$.

■ Let X and Y be two random variables where $E(X) = \mu_X$, $Var(X) = \sigma_X^2$, $E(Y) = \mu_Y$, $Var(Y) = \sigma_Y^2$ and their correlation is ρ. If $E(Y|X) = a + bX$ where a and b are constants, then $a =$ ___(4)___, $b =$ ___(5)___, and $E[Var(Y|X)] =$ ___(6)___.

■ Let X be a random variable with the distribution

$$f(x) = \frac{x^{\alpha-1}}{\Gamma(\alpha)\beta^{\alpha}} \exp\left(-\frac{x}{\beta}\right),$$

where $\alpha > 0$ and $\beta > 0$. If $Y = 1/X$, the probability density function of Y is ___(7)___, and $E(Y^r) =$ ___(8)___, where r is a positive integer.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

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■ Let X be a random variable with the distribution $f(x) = \theta x^{\theta-1}$, where $0 < x < 1$ and $\theta > 0$. Let $E(X) = \mu_x$ and $\widehat{S} = (1/n)\sum_{i=1}^{n} X_i$,where $i = 1, 2, ..., n$. The statistic $\widehat{S}$ converges in probability to ___(9)___, and $\sqrt{n}\left(\widehat{S} - \mu_x\right)$ converges in distribution to ___(10)___.

■ Consider a linear model $y_i = \beta x_i + u_i$ where $u_i|x_i \sim N(0, \sigma_u^2)$. The maximum likelihood estimator $\widehat{\beta} =$ ___(11)___, and the maximized log likelihood in terms of the residual $\widehat{u}_i$ is ___(12)___.

■ Consider a linear model $y_i = \mu + \eta_i$ where $E(\eta_i) = 0$ but $Var(\eta_i)$ and the distribution of η are unknown. The method of moments estimator for μ is ___(13)___, and an appropriate estimator for $Var(\eta_i)$ in this case is ___(14)___.

■ Consider a time series $y_t = 0.5y_{t-1} + \epsilon_t$ where $\epsilon_t \overset{i.i.d.}{\sim} (0, \sigma_\epsilon^2)$. Its long-run variance is ___(15)___, and the first-order autocorrelation function is ___(16)___. Consider another process $y_t = 0.5y_{t-1} + \phi y_{t-2} + \epsilon_t$. The range of ϕ that ensures the stationarity of this process is ___(17)___.

■ The table below displays twenty actual observations of a binary variable Y and the predicted probability $\Pr(Y = 1|X)$ using a set of variable X.

Actual	1	1	0	1	0	0	1	0	0	0
Predicted	0.58	0.42	0.12	0.85	0.72	0.08	0.81	0.24	0.61	0.03

Actual	1	1	0	1	0	0	1	0	0	1
Predicted	0.02	0.75	0.33	0.69	0.38	0.59	0.39	0.27	0.17	0.75

The odds of $Y = 1$ relative to $Y = 0$ is ___(18)___. Given the criterion $\widehat{Y} = 1$ if $\Pr(Y = 1|X) \geq 0.5$ and $\widehat{Y} = 0$ otherwise, the percent of correct prediction is ___(19)___, and the coordinate on the receiver operating characteristic (ROC) curve is ___(20)___.

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