109CC02

國立臺北科技大學 109 學年度碩士班招生考試

系所組別：2220 電子工程系碩士班乙組

第一節　工程數學　試題

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注意事項：
1. 本試題共八題，共 100 分。
2. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

一、Determine whether the following statements are true or false. Justify your answer.

1. (6%) Suppose that **A** is an invertible matrix and **u** is a solution of $\mathbf{Ax} = [5\ 6\ 7\ 8]^T$. The solution of $\mathbf{Ax} = [5\ 6\ 9\ 8]^T$ differs from **u** by 2**p**, where **p** is the third column of $\mathbf{A}^{-1}$.
2. (6%) Let **B** be an $n \times m$ matrix. The pivot columns of the reduced row echelon form of **B** form a basis for the column space of **B**.
3. (6%) Let **C** be a real symmetric matrix. Eigenvectors of **C** that correspond to distinct eigenvalues are orthogonal.

二、(5%) Consider the matrix

$$\mathbf{Q} = \begin{bmatrix} -1 & 2 & 1 & -1 & -2 \\ 2 & -4 & 1 & 5 & 7 \\ 2 & -4 & -3 & 1 & 3 \end{bmatrix}$$

Find a basis for the null space of **Q**.

三、(12%) Find an explicit formula for the sequences defined recursively by

$$r_n = r_{n-1} + 2r_{n-2},\ r_0 = 9 \text{ and } r_1 = 0.$$

四、Let W be the plane with equation $x + y - z = 0$.

1. (6%) Find a basis for W.
2. (9%) Find the distance between the point (1, -2, 2) and the plane W.

五、Let X and Y be continuous random variables with the joint probability density function given by

$$f_{X,Y}(x,y) = \begin{cases} c, & x + y \le 1, x \ge 0, y \ge 0 \\ 0, & \text{otherwise} \end{cases}$$

1. (5%) What is the value of the constant c?
2. (5%) Find the probability $\Pr\{X \le Y\}$.
3. (5%) Find the probability $\Pr\{X + Y \le 0.5\}$.

六、(10%) Let $X_1, X_2, \ldots$ be a sequence of independent and identically distributed Gaussian random variables with mean 100 and variance 100. Let N be a Poisson random variable with mean 1. If N is independent of $X_1, X_2, \ldots$, find the expected value and variance of $R = X_1 + X_2 + \ldots + X_N$.

七、(10%) Consider k identical boxes in which each box contains n balls numbered 1 through n. One ball is drawn from each box at random. What is the probability that m, with $m \in \{1, 2, \ldots, n\}$, is the largest number drawn?

八、(15%) Consider a binary communication system that transmits a bit "0" or "1" in each transmission interval. Suppose that the received signal R can be modeled as

$$R = \begin{cases} S + N, & \text{when a bit "1" is sent} \\ -S + N, & \text{when a bit "0" is sent} \end{cases}$$

where S is a positive number known to the receiver and N is a Gaussian random variable with mean 0 and variance σ^2. If the received signal R is larger than a preassigned value T, then the receiver decides that a bit "1" is sent. Otherwise, the receiver decides that a bit "0" is sent. If a bit "0" is sent with probability p and a bit "1" is sent with probability 1-p, find the probability that the received bit is different form the transmitted bit. Your answer should be expressed in terms of S, T, p, σ, and $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{y^2}{2}}dy$.