國立雲林科技大學 109 學年度 系所：工管系
碩士班招生考試試題 科目：作業研究

(You need to define and explain your notations clearly. Points are granted only if detail computations are presented.)

1. Assume the air pollution indicator, PM2.5 at a city each day can be modeled as a Markov chain with states being: G: green, Y: yellow and R: red. Assume that the state transitions of PM2.5 each day are determined by the following state transition probabilities: P(G|G) =1/3, P(Y|G) = 1/3, P(R|G) = 1/3; P(G|Y) = 1/2, P(Y|Y) =0, P(R|Y) =1/2; P(G|R) = 1/4, P(Y|R) = 1/4, P(R|R) = 1/2;
   a. Find the expected number of days of PM2.5 being R in 360 days. (%15)
   b. Assume the total medical expense related to diseases caused by PM2.5 depends on the state of PM2.5. When the state is R, the medical expense per day is $300; when the state is Y, the medical expense per day is $100 and when the state is G, the medical expense per day is $0. Find the expected medical expense caused by PM2.5 per day in steady state. (%5)

2. Consider a check-out counter at a large supermarket. The arrival stream of customers to the check-out counter is a Poisson process of rate 3 per minute. The service time per check out is exponential with mean time of 0.5 minute. Normally the check-out counter is served by one clerk. When the number of waiting customers exceeds 4, another clerk is added to support the original clerk and these two clerks work together and reduce the mean service time per check out to 0.2 minute per check out. The added clerk leaves the check-out counter at the moment when the number of waiting customers is reduced to 2. Assume that the check-out line can accommodate at most 5 customers (including the one being served).
   a. Define the state of this system and draw the transition rate diagram. (%20)
   b. Find the expected queue length at the check-out counter. (%10)

3. A company sells an item, and the demand over the next 4 months are 100, 140, 210, and 180 units, respectively. The company can stock just enough supply to meet month's demand, or it can overstock to meet the demand for two or more consecutive months. In the latter case, a holding cost of $1.2 is charged per overstocked unit per month. The company estimates the unit purchase prices for the next four months to be $15, $12, $10, and $14, respectively. A setup cost of $200 is incurred each time a purchased order is placed. The company want to develop a purchasing plan that will minimize the total cost of ordering, purchasing, and holding stocks. Formulate the problem as a shortest-route problem. (%25)
   [Hint: You need to define what the nodes express and estimate the distance (cost) between each pair of nodes before modeling.]

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4. Solve the following problem by algebraic Simplex Method. (%15)

$$
\begin{aligned}
&Max\ z = 3x_1 - x_2 + 3x_3 + 4x_4 \\
&s.t. \\
&x_1 + 2x_2 + 2x_3 + 4x_4 \le 40 \\
&2x_1 - 2x_2 + x_3 + 2x_4 \le 8 \\
&4x_1 - 2x_2 + x_3 - x_4 \le 10 \\
&x_1, x_2, x_3, x_4 \ge 0
\end{aligned}
$$

5. Write the dual for the following primal problem. (%10)

$$
\begin{aligned}
&Max\ z = 0.4x_1 + 0.5x_2 \\
&s.t. \\
&0.3x_1 + 0.1x_2 \le 2.7 \\
&0.5x_1 + 0.5x_2 = 6 \\
&0.6x_1 + 0.4x_2 \ge 6 \\
&x_1, x_2 \ge 0
\end{aligned}
$$