



1. (10%) Solving $x \frac{dy}{dx} = x^2 + 3y$
2. (10%) Solve the differential equation $y'' + 4y = 2 \sin x$
3. (10%) Solve the differential equation $y' = -\frac{2xy + \cos x}{x^2 + \sin y}$
4. (20%) A differential equation system $\begin{cases} y_1' = 1y_1 - 3y_2 \\ y_2' = 4y_1 - 6y_2 \end{cases}$ with $y_1(0) = 2$ and $y_2(0) = 1$
 - (a) (10%) Solving the differential equation system.
 - (b) (05%) Plot its phase portrait.
 - (c) (05%) Analyze the stability of this system.
5. (15%) Perform the indicated operation, give that:

$$A = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 2 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 2 & -1 \end{bmatrix} \quad D = \begin{bmatrix} 2 & 2 \\ -1 & 4 \\ 0 & 1 \end{bmatrix}$$

- (a) $(A+B)^T$ (b) $(2A-B)(C+D)$ (c) If $2X - (A-B) = 0$, Find X
6. (15%) Evaluation of these determinants.

$$(a) \begin{vmatrix} \sin n\theta & -\cos n\theta \\ \cos n\theta & \sin n\theta \end{vmatrix} \quad (b) \begin{vmatrix} -2 & 0 & 0 \\ 3 & 1 & 0 \\ 1 & 3 & 5 \end{vmatrix} \quad (c) \begin{vmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{vmatrix}$$

$$7. (10\%) \text{ If } \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}, \text{ Please find } \begin{bmatrix} x \\ y \end{bmatrix}$$

8. (10%) Find the eigenvalues and eigenvectors of A.

$$A = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 4 & 0 \\ -2 & 0 & 5 \end{bmatrix}$$