



1. (15%) Find the general solution of the following differential equations.

$$(a) \frac{dy}{dx} = \frac{3ye^x + 4y \sin xy + 2x^2}{-3e^x - 4x \sin xy} \quad (b) x' + 2x = 4y \quad (c) y'' - 4y' + 4y = 3xe^{-x}$$

2. (15%) Find the Laplace transform or inverse Laplace transform of the following functions.

$$(a) \frac{s+5}{(s+1)^2(s+3)} \quad (b) 5 + 4t^2 + 2\sin 3t \quad (c) \frac{2s+5}{s^2+7} + \frac{3s+1}{s^2-4s+13}$$

3. (10%) Solve the following initial value problem **with Laplace transform**.

$$y'' - 4y' + 4y = e^x, \quad y(0) = 1, \quad y'(0) = 2$$

4. (15%) Using **Gram-Schmidt orthogonalization process**, finding orthogonal matrix **P** from the

given matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 3 & 0 \\ -1 & 0 & 1 \end{bmatrix}$.

5. (10%) Given $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$, please verify whether $\det(\mathbf{AB})$ equals $[\det(\mathbf{A}) \det(\mathbf{B})]$ or not.

6. (15%) Find the Fourier series of $f(x) = \begin{cases} -2, & -\pi < x < 0 \\ 1, & 0 \leq x < \pi \end{cases}$, where $-\pi < x < \pi$.

7. (20%) Solve the following differential equation.

$$x^2 y'' + 2xy' + 2y = x^3 \sin x \quad (x > 0)$$