1.(a) Give the "engineering Bernoulli equation" and explain the physical meaning of each term. (7%)

(b) Using (a) to explain why pressure decreases as an incompressible fluid is flowing along a horizontal pipe. (4%)

2. Consider a single particle settling freely in an incompressible, Newtonian fluid. The particle density and diameter are ρ_p and d_p, respectively. The density and viscosity of the fluid are ρ and μ. ρ_p is greater than ρ.

(a) What is Stokes' law? (5 %)

(b) **Describe** how to determine terminal velocity u_t of the particle. (8%)
Note: State the method only. Evaluation of u_t is not required.

3. Consider a solid sphere of radius R which is rotating in ϕ direction at an angular velocity ω as shown in the following figure. The sphere is immersed in an infinite volume of quiescent fluid. Assume **creeping flow** condition, and $v_\phi = v_\phi(r,\theta)$, $v_r = v_\theta = 0$, $\partial P/\partial\phi = 0$. Here P is the modified pressure.

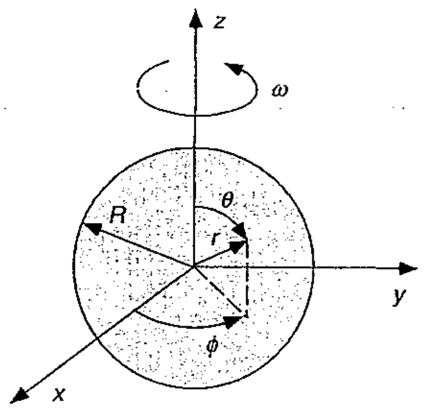
(a) Using the assumptions above, simplify the Navier–Stokes equations to obtain the governing equation for v_ϕ. (5 %)

(b) Provide the boundary conditions required for solving this problem. (6 %)

(c) A reasonable solution form for v_ϕ is $v_\phi = f(r)\sin\theta$. f(r) is a function of r. Explain why this choice is reasonable. (4 %)

(d) Solve for the velocity profile v_ϕ. (6%)

(e) Assuming v_ϕ has been obtained, **provide the formula** to calculate the torque required to maintain the rotation of the sphere. (5%)
Note: State the relevant equations only; evaluation of the integral is not required.



Spherical coordinates (r, θ, ϕ)*:*

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{1}{r^2}\frac{\partial^2}{\partial r^2}(r^2 v_r) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial v_r}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_r}{\partial\phi^2}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta - v_\phi^2\cot\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\theta}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(v_\theta\sin\theta)\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\theta}{\partial\phi^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial\theta} - \frac{2\cot\theta}{r^2\sin\theta}\frac{\partial v_\phi}{\partial\phi}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial v_\phi}{\partial t} + v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_\phi v_r + v_\theta v_\phi\cot\theta}{r}\right) = -\frac{1}{r\sin\theta}\frac{\partial p}{\partial \phi} + \mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(v_\phi\sin\theta)\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\phi}{\partial\phi^2} + \frac{2}{r^2\sin\theta}\frac{\partial v_r}{\partial\phi} + \frac{2\cot\theta}{r^2\sin\theta}\frac{\partial v_\theta}{\partial\phi}\right] + \rho g_\phi$$

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4. A flat solar panel operates at noon under steady-state conditions. The incident solar irradiation is G = 1000 W/m^2. The panel efficiency is η = 20%, and the back side of the panel is perfectly insulated. Heat is removed from the front surface by convection to ambient air at T_∞ = 25°C with a heat-transfer coefficient h = 20 W/(m^2·K). Neglect thermal radiation and heat storage. Estimate the steady-state surface temperature of the solar panel. (10%)

5. A dilute solute A is absorbed from a gas into a liquid (mass transfer direction: gas → liquid). Under the two-film theory, define: y_A: mole fraction of A in the bulk gas; x_A: mole fraction of A in the bulk liquid; $y_{A,i}$ and $x_{A,i}$: interfacial mole fractions of A on the gas and liquid sides.

Assume local interfacial equilibrium with a linear relation: $y_{A,i} = m\, x_{A,i}$, where m is the equilibrium slope.

(a) Derive K_y and K_x in terms of k_y, k_x, and m. (5%)

(b) Explain the physical meaning of $m >> 1$. Which phase controls the overall mass transfer (based on K_y)? (5%)

6. A laser cuts an infinitely long aluminum plate of thickness t = 1 mm. Assume the plate extends to infinity on both sides of the cutting line. Ambient air is at T_∞ = 25°C and h = 50 W/(m^2·K) on the surfaces. Thermal conductivity k = 237 W/(m·K). Cutting line at x = 0 is maintained at T_m = 660°C.

(a) Derive the steady-state temperature distribution $T(x)$. (10%)

(b) Determine the minimum laser power per unit width of the cut. (5%)

7. A vertical Stefan tube (d = 1.0 cm) contains liquid water. Distance to opening L = 10 cm. Top air is 25°C, 1 atm, relative humidity ϕ = 50%. Neglect natural convection. $D_{AB} = 2.6 \times 10^{-5}$ m^2/s. P^{sat} = 3.17 kPa.

(a) Formulate the governing equation for $y_A(z)$ and boundary conditions. (10%)

(b) Estimate the evaporation rate (kg/s). (5%)