國立臺灣大學 115 學年度碩士班招生考試試題

1. Let A and B be two events.

   (a) (5%) If A and B are disjoint, can they be independent? Verify your answer.

   (b) (5%) If $A \subset B$, can A and B be independent? Verify your answer.

2. (10%) Two teams, A and B, play a series of games. If team A has probability .4 of winning each game, is it to its advantage to play the best two out of three games or just one game? Explain why playing a longer or shorter series of game benefit team A. Assume the outcomes of successive games are independent.

3. (10%) The exponential distribution is $f(x;\lambda) = \lambda \exp(-\lambda x)$ and $\mathbb{E}(X) = \lambda^{-1}$. The cdf is $F(x) = \Pr(X \leq x) = 1 - \exp(-\lambda x)$. Three observations are made by an instrument that reports $x_1 = 5$ and $x_2 = 3$, but x_3 is too large for the instrument to measure and it reports only that $x_3 > 10$, i.e., the largest value the instrument can measure is 10. (**Hint**: The x_3 is unknown and should not appear in the likelihood function.)

   (a) What is the likelihood function?

   (b) What is the MLE of λ?

4. (15%) Let $X_1, X_2, \dots$ be a sequence of independent random variables having mean 0, variance σ^2, the common distribution function F and moment-generating function (MGF) M defined in a neighborhood of zero. Let

$$S_n = \sum_{i=1}^{n} X_i.$$

   Use MGF and Taylor expansion to show

$$\lim_{n\to\infty} \Pr\left(\frac{S_n}{\sigma\sqrt{n}} \leq x\right) = \Phi(x),$$

   where Φ is the cdf of $N(0,1)$. The MGF of $N(0,1)$ is $\exp(t^2/2)$.

5. Let $X_1, \dots, X_n$ be i.i.d. samples from a Poisson distribution with parameter λ, i.e., $f(x;\lambda) = \frac{\lambda^x e^{-\lambda}}{x!}$ for $x = 0, 1, 2, \dots$ and $\lambda > 0$.

   (a) (5%) Find the UMVUE of λ.

   (b) (10%) Find the UMVUE of λ^2.

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6. Let $X_1, \ldots, X_n$ be i.i.d. samples from a distribution with pdf $f(x;\theta) = \theta x^{\theta-1}$ for $0 < x < 1$ and $\theta > 0$.

   (a) (10%) Find the Uniform Most Powerful (UMP) test for testing $H_0 : \theta \leq \theta_0$ against $H_1 : \theta > \theta_0$ at significance level α.

   (b) (10%) Find the distribution of the test statistic under H_0 and determine the rejection region.

7. (20%) Let X_1, X_2, X_3 be i.i.d. samples from an exponential distribution with parameter λ. Let $X_{(1)} < X_{(2)} < X_{(3)}$ denote the order statistics. Define the sample spacings as $W_1 = X_{(1)}$, $W_2 = X_{(2)} - X_{(1)}$, and $W_3 = X_{(3)} - X_{(2)}$. Show that W_1, W_2, W_3 are independent and identify their distributions.

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