題號： 235　　國立臺灣大學 115 學年度碩士班招生考試試題
科目： 線性代數(C)
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Q1. Which of the following operations CHANGES the solution set of a linear system?
A. Swapping two rows
B. Adding a multiple of one row to another
C. Multiplying a row by a nonzero scalar
D. Multiplying a row by zero

Q2. If a square matrix has two columns whose sum is the zero vector, its determinant is _____.

Q3. For an n×n matrix A, which condition guarantees that A is invertible?
A. $\det(A) = 0$
B. Columns of A are linearly dependent
C. Rows of A are linearly independent
D. A has a zero entry

Q4. If a square matrix A whose $\det(A)$ is zero, there must exit an eigenvalue equal to:
A. 1
B. $\det(A-I)$
C. $\det(A)$
D. $\det(A^TA-I)$

Q5. A real symmetric matrix always has:
A. Complex eigenvalues only
B. Real eigenvalues
C. Zero determinant
D. Full rank

Q6. Which set forms a basis for $\mathbb{R}$?
A. {(1,0),(2,0)}
B. {(1,1),(−1,−1)}
C. {(1,0),(1,1)}
D. {(0,0),(1,1)}

Q7. If vectors u and v are orthonormal, then:
A. $u \cdot v = 1$
B. $\|u\| = \|v\| = 1$
C. $u = v$
D. They are linearly dependent

Q8. If a matrix is positive definite, it must NOT be:
A. Diagonal
B. Symmetric
C. Singular
D. Orthogonal

Q9. Which elementary row operation changes the sign of $\det(A)$?
A. Multiply a row by a nonzero scalar
B. Add a multiple of one row to another
C. Swap two rows
D. Transpose A

Q10. Which condition ensures that $Ax=b$ has a unique solution for every b in $\mathbb{R}^n$ (A is $n\times n$)?
A. A has more equations than unknowns
B. A has linearly dependent rows
C. A has linearly independent columns
D. A has no zero entry

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Q11. If $x^TAx > 0$ for all nonzero x, then A is:

A. Negative definite

B. Positive definite

C. Singular

D. Orthogonal

Q12. If v_1 and v_2 are linearly independent vectors in $\mathbb{R}$, then:

A. They are orthogonal

B. They span $\mathbb{R}$

C. One is a scalar multiple of the other

D. They must be eigenvectors

Q13. Eigenvectors corresponding to distinct eigenvalues are always:

A. Orthogonal

B. Linearly independent

C. Equal

D. Linearly dependent

Q14. If A is positive definite, then det(A) is _____.

Q15. Which matrix is positive definite?

A. [[1,0],[0,–1]]

B. [[2,1],[1,2]]

C. [[0,1],[1,0]]

D. [[1,2],[2,4]]

Q16. If A is real symmetric, then A is diagonalizable by an:

A. Arbitrary invertible matrix

B. Lower-triangular matrix

C. Orthogonal matrix

D. Permutation matrix only

Q17. Which statement is always true for orthogonal matrices Q (real)?

A. All eigenvalues are positive

B. Columns of Q form an orthonormal basis

C. det(Q)=1 always

D. Q is symmetric

Q18. If an $n \times n$ matrix A has n distinct eigenvalues, then A is:

A. Defective

B. Not diagonalizable

C. Diagonalizable

D. Invertible

Q19. For real symmetric $n \times n$ matrix A, eigenvectors from n different eigenvalues can NOT be:

A. Orthogonal

B. Summed to zero

C. Linearly independent

D. Spanning $\mathbb{R}^n$

Q20. If A is real symmetric positive definite, which factorization always exists?

A. LU with no row exchanges for all such A

B. QR with R having negative diagonal

C. Cholesky $A=LL^T$

D. Jordan form with one block

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Q21. Which condition is equivalent to A being positive definite (real symmetric A)?

A. All diagonal entries are positive

B. All eigenvalues are positive

C. $\det(A)>0$

D. A is invertible

Q22. If A and B are similar matrices, then they have the same _____.

Q23. For real symmetric A, x^TAx subject to $\|x\|=1$ is minimized by choosing x as an eigenvector of A for the:

A. Largest eigenvalue

B. Smallest eigenvalue

C. Zero eigenvalue

D. Trace of A

Q24. If A is real symmetric indefinite, then:

A. All eigenvalues are positive

B. All eigenvalues are negative

C. A has both positive and negative eigenvalues

D. $\det(A)<0$

Q25. If A has a repeated eigenvalue, then:

A. A is never diagonalizable

B. A must be defective

C. A may or may not be diagonalizable

D. Eigenvectors are unique

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