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| 考 試 科 目 | 統計學 | 系 所 別 | 財政學系 | 考 試 時 間 | 2 月 5 日(四) 第 4 節 |
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說明：

- 作答時請完整列出過程，評分同時考量答案完整性及正確性
- 查表過程中，若本試題所提供之統計附表無對應數值，請使用最接近之查表數值代替。

Question 1 (15%)

Let X and Y be two random variables representing monthly asset returns for two assets. You are given the following summary statistics:

- Sample Variance of X : $S_x^2 = 4$
- Sample Variance of Y : $S_y^2 = 9$
- Sample Covariance: $S_{xy} = -3$

- (7%) Compute the Pearson correlation coefficient (r_{xy}) between X and Y .
- (8%) An analyst proposes a linear transformation of the original variables to account for leverage and fees. The new variables are defined as:

$$W = 3X + 5$$

$$Z = -2Y + 1$$

Derive the value of the covariance between W and Z (i.e., S_{wz}).

Question 2 (20%)

You are advising a regulator on the solvency of a bank. Let S denote "Solvent" and I denote "Insolvent". Based on historical market data, the prior probability of the bank being Insolvent is $P(I) = 0.05$.

You have access to two distinct, conditionally independent auditing algorithms: Audit A and Audit B. If the bank is Insolvent, Audit A flags it positive (A^+) with probability 0.90. If the bank is Solvent, Audit A flags it positive (A^+) with probability 0.10. If the bank is Insolvent, Audit B flags it positive (B^+) with probability 0.80. If the bank is Solvent, Audit B flags it positive (B^+) with probability 0.05.

- (6%) Suppose Audit A returns a positive flag (A^+). Calculate the posterior probability that the bank is **Insolvent**.
- (8%) After observing the positive result from Audit A in (a), you run Audit B and it also returns a positive flag (B^+). Update your posterior probability that the bank is **Insolvent** after this observation.
- (6%) The regulator considers intervening. Intervention incurs costs C . If the bank is truly Insolvent, intervention prevents a financial loss L . If the bank is Solvent, intervention provides no benefit.
 - Express the condition under which the regulator should intervene in terms of the posterior probability of insolvency, π^* , and the ratio C/L .
 - Based on your answer in (b), if the potential loss L is **3 times** the cost of intervention C , should the regulator intervene?

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- 二、試題請隨卷繳交。

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Question 3 (20%)

A Public Health Department is evaluating compliance with a new "Sugar Tax" regulation. Based on preliminary data, the probability that a randomly selected retailer is non-compliant is 20%.

You randomly select a sample of $n = 100$ retailers for inspection. Let X denote the number of non-compliant retailers in your sample.

- (5%) State the expected value and the variance of the random variable X .
- (5%) Write the expression for the probability of finding exactly 25 non-compliant retailers using the Binomial PMF. (You do not need to compute the final decimal value).
- (5%) Use the Normal Approximation to estimate the probability that the number of non-compliant retailers is strictly greater than 25 (i.e., $P(X > 25)$). Is this approximation appropriate given the sample size and proportion? Briefly explain.
- (5%) A junior analyst suggests that if the sample size were extremely large ($n \rightarrow \infty$) and the probability of non-compliance were extremely small ($p \rightarrow 0$) such that np remained constant, a different distribution should be used. Which distribution is the analyst referring to?

Question 4 (20%)

A public agency pilots three training programs for unemployed workers: Program A, Program B, and Program C. The outcome is post-training monthly earnings (in ten-thousand dollars). Independent samples of participants were selected from each program.

You observe the following summary statistics:

| Program | Sample Size | Sample Mean | Sample Variance |
|---------|-------------|-------------|-----------------|
| A | 4 | 2.1 | 0.36 |
| B | 4 | 2.6 | 0.49 |
| C | 4 | 1.9 | 0.25 |

- (12%) Construct an ANOVA table to partition the variance in monthly earnings.
- (8%) Conduct an F-test at the 5% level. Based on the results, what can you conclude about the relative effectiveness of these programs?

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Question 5 (25%)

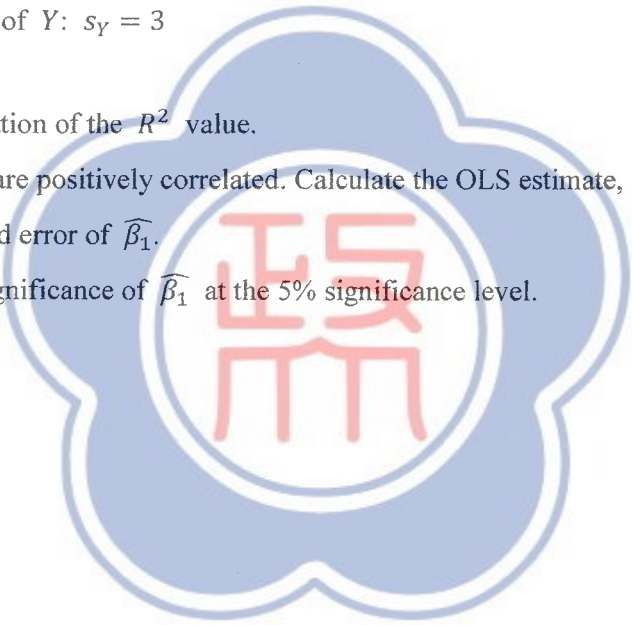
You study whether household annual consumption expenditure Y_i (in thousands of dollars) is related to household disposable income X_i (in thousands of dollars) using a cross-sectional survey of $n = 50$ households. You estimate the OLS regression with an intercept:

$$Y_i = \beta_0 + \beta_1 X_i + u_i, \quad i = 1, \dots, 50$$

Assume the observations are independent, and the error term (u_i) is homoskedastic and normally distributed.

From the regression output and sample summaries:

- $R^2 = 0.36$
 - Sample standard deviation of X : $s_X = 2$
 - Sample standard deviation of Y : $s_Y = 3$
- a) (5%) Provide an interpretation of the R^2 value.
 - b) (7%) Suppose X and Y are positively correlated. Calculate the OLS estimate, $\widehat{\beta}_1$.
 - c) (8%) Compute the standard error of $\widehat{\beta}_1$.
 - d) (5%) Test the statistical significance of $\widehat{\beta}_1$ at the 5% significance level.



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Cumulative Probabilities for Standard Normal Distribution

| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |

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Student's t-Distribution

| Degrees of Freedom | | Area in Upper Tail | | | | | Degrees of Freedom | | Area in Upper Tail | | | | |
|--------------------|-------|--------------------|-------|--------|--------|--------|--------------------|-------|--------------------|-------|-------|-------|-------|
| Freedom | 0.2 | 0.1 | 0.05 | 0.025 | 0.01 | 0.005 | Freedom | 0.2 | 0.1 | 0.05 | 0.025 | 0.01 | 0.005 |
| 1 | 1.376 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 | 51 | 0.849 | 1.298 | 1.675 | 2.008 | 2.402 | 2.676 |
| 2 | 1.061 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 52 | 0.849 | 1.298 | 1.675 | 2.007 | 2.400 | 2.674 |
| 3 | 0.978 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 53 | 0.848 | 1.298 | 1.674 | 2.006 | 2.399 | 2.672 |
| 4 | 0.941 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 54 | 0.848 | 1.297 | 1.674 | 2.005 | 2.397 | 2.670 |
| 5 | 0.920 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 55 | 0.848 | 1.297 | 1.673 | 2.004 | 2.396 | 2.668 |
| 6 | 0.906 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 56 | 0.848 | 1.297 | 1.673 | 2.003 | 2.395 | 2.667 |
| 7 | 0.896 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 57 | 0.848 | 1.297 | 1.672 | 2.002 | 2.394 | 2.665 |
| 8 | 0.889 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 58 | 0.848 | 1.296 | 1.672 | 2.002 | 2.392 | 2.663 |
| 9 | 0.883 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 59 | 0.848 | 1.296 | 1.671 | 2.001 | 2.391 | 2.662 |
| 10 | 0.879 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 60 | 0.848 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 |
| 11 | 0.876 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 61 | 0.848 | 1.296 | 1.670 | 2.000 | 2.389 | 2.659 |
| 12 | 0.873 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 62 | 0.847 | 1.295 | 1.670 | 1.999 | 2.388 | 2.657 |
| 13 | 0.870 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 63 | 0.847 | 1.295 | 1.669 | 1.998 | 2.387 | 2.656 |
| 14 | 0.868 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 64 | 0.847 | 1.295 | 1.669 | 1.998 | 2.386 | 2.655 |
| 15 | 0.866 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 65 | 0.847 | 1.295 | 1.669 | 1.997 | 2.385 | 2.654 |
| 16 | 0.865 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 66 | 0.847 | 1.295 | 1.668 | 1.997 | 2.384 | 2.652 |
| 17 | 0.863 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 67 | 0.847 | 1.294 | 1.668 | 1.996 | 2.383 | 2.651 |
| 18 | 0.862 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 68 | 0.847 | 1.294 | 1.668 | 1.995 | 2.382 | 2.650 |
| 19 | 0.861 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 69 | 0.847 | 1.294 | 1.667 | 1.995 | 2.382 | 2.649 |
| 20 | 0.860 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 70 | 0.847 | 1.294 | 1.667 | 1.994 | 2.381 | 2.648 |
| 21 | 0.859 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 71 | 0.847 | 1.294 | 1.667 | 1.994 | 2.380 | 2.647 |
| 22 | 0.858 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 72 | 0.847 | 1.293 | 1.666 | 1.993 | 2.379 | 2.646 |
| 23 | 0.858 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 73 | 0.847 | 1.293 | 1.666 | 1.993 | 2.379 | 2.645 |
| 24 | 0.857 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 74 | 0.847 | 1.293 | 1.666 | 1.993 | 2.378 | 2.644 |
| 25 | 0.856 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 75 | 0.846 | 1.293 | 1.665 | 1.992 | 2.377 | 2.643 |
| 26 | 0.856 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 76 | 0.846 | 1.293 | 1.665 | 1.992 | 2.376 | 2.642 |
| 27 | 0.855 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 77 | 0.846 | 1.293 | 1.665 | 1.991 | 2.376 | 2.641 |
| 28 | 0.855 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 78 | 0.846 | 1.292 | 1.665 | 1.991 | 2.375 | 2.640 |
| 29 | 0.854 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 79 | 0.846 | 1.292 | 1.664 | 1.990 | 2.374 | 2.640 |
| 30 | 0.854 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 | 80 | 0.846 | 1.292 | 1.664 | 1.990 | 2.374 | 2.639 |
| 31 | 0.853 | 1.309 | 1.696 | 2.040 | 2.453 | 2.744 | 81 | 0.846 | 1.292 | 1.664 | 1.990 | 2.373 | 2.638 |
| 32 | 0.853 | 1.309 | 1.694 | 2.037 | 2.449 | 2.738 | 82 | 0.846 | 1.292 | 1.664 | 1.989 | 2.373 | 2.637 |
| 33 | 0.853 | 1.308 | 1.692 | 2.035 | 2.445 | 2.733 | 83 | 0.846 | 1.292 | 1.663 | 1.989 | 2.372 | 2.636 |
| 34 | 0.852 | 1.307 | 1.691 | 2.032 | 2.441 | 2.728 | 84 | 0.846 | 1.292 | 1.663 | 1.989 | 2.372 | 2.636 |
| 35 | 0.852 | 1.306 | 1.690 | 2.030 | 2.438 | 2.724 | 85 | 0.846 | 1.292 | 1.663 | 1.988 | 2.371 | 2.635 |
| 36 | 0.852 | 1.306 | 1.688 | 2.028 | 2.434 | 2.719 | 86 | 0.846 | 1.291 | 1.663 | 1.988 | 2.370 | 2.634 |
| 37 | 0.851 | 1.305 | 1.687 | 2.026 | 2.431 | 2.715 | 87 | 0.846 | 1.291 | 1.663 | 1.988 | 2.370 | 2.634 |
| 38 | 0.851 | 1.304 | 1.686 | 2.024 | 2.429 | 2.712 | 88 | 0.846 | 1.291 | 1.662 | 1.987 | 2.369 | 2.633 |
| 39 | 0.851 | 1.304 | 1.685 | 2.023 | 2.426 | 2.708 | 89 | 0.846 | 1.291 | 1.662 | 1.987 | 2.369 | 2.632 |
| 40 | 0.851 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 | 90 | 0.846 | 1.291 | 1.662 | 1.987 | 2.368 | 2.632 |
| 41 | 0.850 | 1.303 | 1.683 | 2.020 | 2.421 | 2.701 | 91 | 0.846 | 1.291 | 1.662 | 1.986 | 2.368 | 2.631 |
| 42 | 0.850 | 1.302 | 1.682 | 2.018 | 2.418 | 2.698 | 92 | 0.846 | 1.291 | 1.662 | 1.986 | 2.368 | 2.630 |
| 43 | 0.850 | 1.302 | 1.681 | 2.017 | 2.416 | 2.695 | 93 | 0.846 | 1.291 | 1.661 | 1.986 | 2.367 | 2.630 |
| 44 | 0.850 | 1.301 | 1.680 | 2.015 | 2.414 | 2.692 | 94 | 0.845 | 1.291 | 1.661 | 1.986 | 2.367 | 2.629 |
| 45 | 0.850 | 1.301 | 1.679 | 2.014 | 2.412 | 2.690 | 95 | 0.845 | 1.291 | 1.661 | 1.985 | 2.366 | 2.629 |
| 46 | 0.850 | 1.300 | 1.679 | 2.013 | 2.410 | 2.687 | 96 | 0.845 | 1.290 | 1.661 | 1.985 | 2.366 | 2.628 |
| 47 | 0.849 | 1.300 | 1.678 | 2.012 | 2.408 | 2.685 | 97 | 0.845 | 1.290 | 1.661 | 1.985 | 2.365 | 2.627 |
| 48 | 0.849 | 1.299 | 1.677 | 2.011 | 2.407 | 2.682 | 98 | 0.845 | 1.290 | 1.661 | 1.984 | 2.365 | 2.627 |
| 49 | 0.849 | 1.299 | 1.677 | 2.010 | 2.405 | 2.680 | 99 | 0.845 | 1.290 | 1.660 | 1.984 | 2.365 | 2.626 |
| 50 | 0.849 | 1.299 | 1.676 | 2.009 | 2.403 | 2.678 | 100 | 0.845 | 1.290 | 1.660 | 1.984 | 2.364 | 2.626 |

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F-Distribution

| Denominator Degrees of freedom | Area in Upper Tail | Numerator Degrees of Freedom | | | | | | | | | | | | | | |
|--------------------------------|--------------------|------------------------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|--|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 15 | 20 | 25 | 30 | |
| 1 | 0.100 | 39.86 | 49.50 | 53.59 | 55.83 | 57.24 | 58.20 | 58.91 | 59.44 | 59.86 | 60.19 | 61.22 | 61.74 | 62.05 | 62.26 | |
| | 0.050 | 161.45 | 199.50 | 215.71 | 224.58 | 230.16 | 233.99 | 236.77 | 238.88 | 240.54 | 241.88 | 245.95 | 248.01 | 249.26 | 250.10 | |
| | 0.025 | 647.79 | 799.50 | 864.16 | 899.58 | 921.85 | 937.11 | 948.22 | 956.66 | 963.28 | 968.63 | 984.87 | 993.10 | 998.08 | 1001.41 | |
| | 0.010 | 4052.18 | 4999.50 | 5403.35 | 5624.58 | 5763.65 | 5858.99 | 5928.36 | 5981.07 | 6022.47 | 6055.85 | 6157.28 | 6208.73 | 6239.83 | 6260.65 | |
| | 0.100 | 8.53 | 9.00 | 9.16 | 9.24 | 9.29 | 9.33 | 9.35 | 9.37 | 9.38 | 9.39 | 9.42 | 9.44 | 9.45 | 9.46 | |
| 2 | 0.100 | 18.51 | 19.00 | 19.16 | 19.25 | 19.30 | 19.33 | 19.35 | 19.37 | 19.38 | 19.40 | 19.43 | 19.45 | 19.46 | 19.46 | |
| | 0.050 | 38.51 | 39.00 | 39.17 | 39.25 | 39.30 | 39.33 | 39.36 | 39.37 | 39.39 | 39.40 | 39.43 | 39.45 | 39.46 | 39.46 | |
| | 0.025 | 98.50 | 99.00 | 99.17 | 99.25 | 99.30 | 99.33 | 99.36 | 99.37 | 99.39 | 99.40 | 99.43 | 99.45 | 99.46 | 99.47 | |
| | 0.010 | 98.50 | 99.00 | 99.17 | 99.25 | 99.30 | 99.33 | 99.36 | 99.37 | 99.39 | 99.40 | 99.43 | 99.45 | 99.46 | 99.47 | |
| | 0.100 | 5.54 | 5.46 | 5.39 | 5.34 | 5.31 | 5.28 | 5.27 | 5.25 | 5.24 | 5.23 | 5.20 | 5.18 | 5.17 | 5.17 | |
| 3 | 0.100 | 10.13 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 | 8.81 | 8.79 | 8.70 | 8.66 | 8.63 | 8.62 | |
| | 0.050 | 17.44 | 16.04 | 15.44 | 15.10 | 14.88 | 14.73 | 14.62 | 14.54 | 14.47 | 14.42 | 14.25 | 14.17 | 14.12 | 14.08 | |
| | 0.025 | 34.12 | 30.82 | 29.46 | 28.71 | 28.24 | 27.91 | 27.67 | 27.49 | 27.35 | 27.23 | 26.87 | 26.69 | 26.58 | 26.50 | |
| | 0.010 | 34.12 | 30.82 | 29.46 | 28.71 | 28.24 | 27.91 | 27.67 | 27.49 | 27.35 | 27.23 | 26.87 | 26.69 | 26.58 | 26.50 | |
| | 0.100 | 4.54 | 4.32 | 4.19 | 4.11 | 4.05 | 4.01 | 3.98 | 3.95 | 3.94 | 3.92 | 3.87 | 3.84 | 3.83 | 3.82 | |
| 4 | 0.100 | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 | 6.00 | 5.96 | 5.86 | 5.80 | 5.77 | 5.75 | |
| | 0.050 | 12.22 | 10.65 | 9.98 | 9.60 | 9.36 | 9.20 | 9.07 | 8.98 | 8.90 | 8.84 | 8.66 | 8.56 | 8.50 | 8.46 | |
| | 0.025 | 21.20 | 18.00 | 16.69 | 15.98 | 15.52 | 15.21 | 14.98 | 14.80 | 14.66 | 14.55 | 14.20 | 14.02 | 13.91 | 13.84 | |
| | 0.010 | 21.20 | 18.00 | 16.69 | 15.98 | 15.52 | 15.21 | 14.98 | 14.80 | 14.66 | 14.55 | 14.20 | 14.02 | 13.91 | 13.84 | |
| | 0.100 | 4.06 | 3.78 | 3.62 | 3.52 | 3.45 | 3.40 | 3.37 | 3.34 | 3.32 | 3.30 | 3.24 | 3.21 | 3.19 | 3.17 | |
| 5 | 0.100 | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 | 4.74 | 4.62 | 4.56 | 4.52 | 4.50 | |
| | 0.050 | 10.01 | 8.43 | 7.76 | 7.39 | 7.15 | 6.98 | 6.85 | 6.76 | 6.68 | 6.62 | 6.43 | 6.33 | 6.27 | 6.23 | |
| | 0.025 | 16.26 | 13.27 | 12.06 | 11.39 | 10.97 | 10.67 | 10.46 | 10.29 | 10.16 | 10.05 | 9.72 | 9.55 | 9.45 | 9.38 | |
| | 0.010 | 16.26 | 13.27 | 12.06 | 11.39 | 10.97 | 10.67 | 10.46 | 10.29 | 10.16 | 10.05 | 9.72 | 9.55 | 9.45 | 9.38 | |
| | 0.100 | 3.78 | 3.46 | 3.29 | 3.18 | 3.11 | 3.05 | 3.01 | 2.98 | 2.96 | 2.94 | 2.87 | 2.84 | 2.81 | 2.80 | |
| 6 | 0.100 | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.10 | 4.06 | 3.94 | 3.87 | 3.83 | 3.81 | |
| | 0.050 | 8.81 | 7.26 | 6.60 | 6.23 | 5.99 | 5.82 | 5.70 | 5.60 | 5.52 | 5.46 | 5.27 | 5.17 | 5.11 | 5.07 | |
| | 0.025 | 13.75 | 10.92 | 9.78 | 9.15 | 8.75 | 8.47 | 8.26 | 8.10 | 7.98 | 7.87 | 7.56 | 7.40 | 7.30 | 7.23 | |
| | 0.010 | 13.75 | 10.92 | 9.78 | 9.15 | 8.75 | 8.47 | 8.26 | 8.10 | 7.98 | 7.87 | 7.56 | 7.40 | 7.30 | 7.23 | |
| | 0.100 | 3.59 | 3.26 | 3.07 | 2.96 | 2.88 | 2.83 | 2.78 | 2.75 | 2.72 | 2.70 | 2.63 | 2.59 | 2.57 | 2.56 | |
| 7 | 0.100 | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 | 3.68 | 3.64 | 3.51 | 3.44 | 3.40 | 3.38 | |
| | 0.050 | 8.07 | 6.54 | 5.89 | 5.52 | 5.29 | 5.12 | 4.99 | 4.90 | 4.82 | 4.76 | 4.57 | 4.47 | 4.40 | 4.36 | |
| | 0.025 | 12.25 | 9.55 | 8.45 | 7.85 | 7.46 | 7.19 | 6.99 | 6.84 | 6.72 | 6.62 | 6.31 | 6.16 | 6.06 | 5.99 | |
| | 0.010 | 12.25 | 9.55 | 8.45 | 7.85 | 7.46 | 7.19 | 6.99 | 6.84 | 6.72 | 6.62 | 6.31 | 6.16 | 6.06 | 5.99 | |
| | 0.100 | 3.46 | 3.11 | 2.92 | 2.81 | 2.73 | 2.67 | 2.62 | 2.59 | 2.56 | 2.54 | 2.46 | 2.42 | 2.40 | 2.38 | |
| 8 | 0.100 | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.39 | 3.35 | 3.22 | 3.15 | 3.11 | 3.08 | |
| | 0.050 | 7.57 | 6.06 | 5.42 | 5.05 | 4.82 | 4.65 | 4.53 | 4.43 | 4.36 | 4.30 | 4.10 | 4.00 | 3.94 | 3.89 | |
| | 0.025 | 11.26 | 8.65 | 7.59 | 7.01 | 6.63 | 6.37 | 6.18 | 6.03 | 5.91 | 5.81 | 5.52 | 5.36 | 5.26 | 5.20 | |
| | 0.010 | 11.26 | 8.65 | 7.59 | 7.01 | 6.63 | 6.37 | 6.18 | 6.03 | 5.91 | 5.81 | 5.52 | 5.36 | 5.26 | 5.20 | |
| | 0.100 | 3.36 | 3.01 | 2.81 | 2.69 | 2.61 | 2.55 | 2.51 | 2.47 | 2.44 | 2.42 | 2.34 | 2.30 | 2.27 | 2.25 | |
| 9 | 0.100 | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.18 | 3.14 | 3.01 | 2.94 | 2.89 | 2.86 | |
| | 0.050 | 7.21 | 5.71 | 5.08 | 4.72 | 4.48 | 4.32 | 4.20 | 4.10 | 4.03 | 3.96 | 3.77 | 3.67 | 3.60 | 3.56 | |
| | 0.025 | 10.56 | 8.02 | 6.99 | 6.42 | 6.06 | 5.80 | 5.61 | 5.47 | 5.35 | 5.26 | 4.96 | 4.81 | 4.71 | 4.65 | |
| | 0.010 | 10.56 | 8.02 | 6.99 | 6.42 | 6.06 | 5.80 | 5.61 | 5.47 | 5.35 | 5.26 | 4.96 | 4.81 | 4.71 | 4.65 | |
| | 0.100 | 3.29 | 2.92 | 2.73 | 2.61 | 2.52 | 2.46 | 2.41 | 2.38 | 2.35 | 2.32 | 2.24 | 2.20 | 2.17 | 2.16 | |
| 10 | 0.100 | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 3.02 | 2.98 | 2.85 | 2.77 | 2.73 | 2.70 | |
| | 0.050 | 6.94 | 5.46 | 4.83 | 4.47 | 4.24 | 4.07 | 3.95 | 3.85 | 3.78 | 3.72 | 3.52 | 3.42 | 3.35 | 3.31 | |
| | 0.025 | 10.04 | 7.56 | 6.55 | 5.99 | 5.64 | 5.39 | 5.20 | 5.06 | 4.94 | 4.85 | 4.56 | 4.41 | 4.31 | 4.25 | |
| | 0.010 | 10.04 | 7.56 | 6.55 | 5.99 | 5.64 | 5.39 | 5.20 | 5.06 | 4.94 | 4.85 | 4.56 | 4.41 | 4.31 | 4.25 | |
| | 0.100 | 3.23 | 2.86 | 2.66 | 2.54 | 2.45 | 2.39 | 2.34 | 2.30 | 2.27 | 2.25 | 2.17 | 2.12 | 2.10 | 2.08 | |
| 11 | 0.100 | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 | 2.95 | 2.90 | 2.85 | 2.72 | 2.65 | 2.60 | 2.57 | |
| | 0.050 | 6.72 | 5.26 | 4.63 | 4.28 | 4.04 | 3.88 | 3.76 | 3.66 | 3.59 | 3.53 | 3.33 | 3.23 | 3.16 | 3.12 | |
| | 0.025 | 9.65 | 7.21 | 6.22 | 5.67 | 5.32 | 5.07 | 4.89 | 4.74 | 4.63 | 4.54 | 4.25 | 4.10 | 4.01 | 3.94 | |
| | 0.010 | 9.65 | 7.21 | 6.22 | 5.67 | 5.32 | 5.07 | 4.89 | 4.74 | 4.63 | 4.54 | 4.25 | 4.10 | 4.01 | 3.94 | |
| | 0.100 | 3.18 | 2.81 | 2.61 | 2.48 | 2.39 | 2.33 | 2.28 | 2.24 | 2.21 | 2.19 | 2.10 | 2.06 | 2.03 | 2.01 | |
| 12 | 0.100 | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.80 | 2.75 | 2.62 | 2.54 | 2.50 | 2.47 | |
| | 0.050 | 6.55 | 5.10 | 4.47 | 4.12 | 3.89 | 3.73 | 3.61 | 3.51 | 3.44 | 3.37 | 3.18 | 3.07 | 3.01 | 2.96 | |
| | 0.025 | 9.33 | 6.93 | 5.95 | 5.41 | 5.06 | 4.82 | 4.64 | 4.50 | 4.39 | 4.30 | 4.01 | 3.86 | 3.76 | 3.70 | |
| | 0.010 | 9.33 | 6.93 | 5.95 | 5.41 | 5.06 | 4.82 | 4.64 | 4.50 | 4.39 | 4.30 | 4.01 | 3.86 | 3.76 | 3.70 | |
| | 0.100 | 3.14 | 2.76 | 2.56 | 2.43 | 2.35 | 2.28 | 2.23 | 2.20 | 2.16 | 2.14 | 2.05 | 2.01 | 1.98 | 1.96 | |
| 13 | 0.100 | 4.67 | 3.81 | 3.41 | 3.18 | 3.03 | 2.92 | 2.83 | 2.77 | 2.71 | 2.67 | 2.53 | 2.46 | 2.41 | 2.38 | |
| | 0.050 | 6.41 | 4.97 | 4.35 | 4.00 | 3.77 | 3.60 | 3.48 | 3.39 | 3.31 | 3.25 | 3.05 | 2.95 | 2.88 | 2.84 | |
| | 0.025 | 9.07 | 6.70 | 5.74 | 5.21 | 4.86 | 4.62 | 4.44 | 4.30 | 4.19 | 4.10 | 3.82 | 3.66 | 3.57 | 3.51 | |
| | 0.010 | 9.07 | 6.70 | 5.74 | 5.21 | 4.86 | 4.62 | 4.44 | 4.30 | 4.19 | 4.10 | 3.82 | 3.66 | 3.57 | 3.51 | |
| | 0.100 | 3.10 | 2.73 | 2.52 | 2.39 | 2.31 | 2.24 | 2.19 | 2.15 | 2.12 | 2.10 | 2.01 | 1.96 | 1.93 | 1.91 | |
| 14 | 0.100 | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 | 2.85 | 2.76 | 2.70 | 2.65 | 2.60 | 2.46 | 2.39 | 2.34 | 2.31 | |
| | 0.050 | 6.30 | 4.86 | 4.24 | 3.89 | 3.66 | 3.50 | 3.38 | 3.29 | 3.21 | 3.15 | 2.95 | 2.84 | 2.78 | 2.73 | |
| | 0.025 | 8.86 | 6.51 | 5.56 | 5.04 | 4.69 | 4.46 | 4.28 | 4.14 | 4.03 | 3.94 | 3.66 | 3.51 | 3.41 | 3.35 | |
| | 0.010 | 8.86 | 6.51 | 5.56 | 5.04 | 4.69 | 4.46 | 4.28 | 4.14 | 4.03 | 3.94 | 3.66 | 3.51 | 3.41 | 3.35 | |
| | 0.100 | 3.07 | 2.70 | 2.49 | 2.36 | 2.27 | 2.21 | 2.16 | 2.12 | 2.09 | 2.06 | 1.97 | 1.92 | 1.89 | 1.87 | |
| 15 | 0.100 | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 | 2.54 | 2.40 | 2.33 | 2.28 | 2.25 | |
| | 0.050 | 6.20 | 4.77 | 4.15 | 3.80 | 3.58 | 3.41 | 3.29 | 3.20 | 3.12 | 3.06 | 2.86 | 2.76 | 2.69 | 2.64 | |
| | 0.025 | 8.68 | 6.36 | 5.42 | 4.89 | 4.56 | 4.32 | 4.14 | 4.00 | 3.89 | 3.80 | 3.52 | 3.37 | 3.28 | 3.21 | |
| | 0.010 | 8.68 | 6.36 | 5.42 | 4.89 | 4.56 | 4.32 | 4.14 | 4.00 | 3.89 | 3.80 | 3.52 | 3.37 | 3.28 | 3.21 | |
| | 0.100 | 3.05 | 2.67 | 2.46 | 2.33 | 2.24 | 2.18 | 2.13 | 2.09 | 2.06 | 2.03 | 1.94 | 1.89 | 1.86 | 1.84 | |
| 16 | 0.100 | 4.49 | 3.63 | 3.24 | 3.01 | 2.85 | 2.74 | 2.66 | 2.59 | 2.54 | 2.49 | 2.35 | 2.28 | 2.23 | 2.19 | |
| | 0.050 | 6.12 | 4.69 | 4.08 | 3.73 | 3.50 | 3.34 | 3.22 | 3.12 | 3.05 | 2.99 | 2.79 | 2.68 | 2.61 | 2.57 | |
| | 0.025 | 8.53 | 6.23 | 5.29 | 4.77 | 4.44 | 4.20 | 4.03 | 3.89 | 3.78 | 3.69 | 3.41 | 3.26 | 3.16 | 3.10 | |
| | 0.010 | 8.53 | 6.23 | 5.29 | 4.77 | 4.44 | 4.20 | 4.03 | 3.89 | 3.78 | 3.69 | 3.41 | 3.26 | 3.16 | 3.10 | |
| | 0.100 | 3.03 | 2.64 | 2.44 | 2.31 | 2.22 | 2.15 | 2.10 | 2.06 | 2.03 | 2.00 | 1.91 | 1.86 | 1.83 | 1.81 | |
| 17 | 0.100 | 4.45 | 3.59 | 3.20 | 2.96 | 2.81 | 2.70 | 2.61 | 2.55 | 2.49 | 2.45 | 2.31 | 2.23 | 2.18 | 2.15 | |
| | 0.050 | 6.04 | 4.62 | 4.01 | 3.66 | 3.44 | 3.28 | 3.16 | 3.06</ | | | | | | | |