本考試禁止使用任何計算機。符號定義如下：
m_e is the rest mass of an electron; c is speed of light; h is Planck constant; k_B is Boltzmann constant; T denotes temperature; λ denotes wavelength; μ denotes chemical potential; E denotes energy.

第一大題：單一選擇題 (single-answer multiple-choice questions)

共 8 題，請於試卷首頁之「選擇題作答區」依題號作答，無需列出計算過程，計算過程不計分。答對一題得 8 分，答錯一題倒扣 2 分，空白 (未答題) 不計分。

1. When a photon undergoes elastic collision with a stationary electron in Compton scattering, what is the relationship between the wavelength shift $\Delta\lambda$ of the scattered photon and the scattering angle θ?
   (A) $\Delta\lambda = h(m_e c)^{-1}(1+\cos\theta)$
   (B) $\Delta\lambda = h(m_e c^2)^{-1}(1+\cos\theta)$
   (C) $\Delta\lambda = 2h(m_e c^2)^{-1}\cos\theta$
   (D) $\Delta\lambda = h(m_e c^2)^{-1}\sin(\theta/2)$
   (E) $\Delta\lambda = 2h(m_e c)^{-1}\sin^2(\theta/2)$
2. About the behavior of Fermi-Dirac distribution function $f(E)$ in the limit $T \to 0$, which of the following is correct?
   (A) $f(E) = 1/2$ for all energy levels
   (B) $f(E) = 1$ when $E < \mu$, and $f(E) = 0$ when $E > \mu$.
   (C) $f(E)$ exhibits a Gaussian distribution
   (D) $f(E)$ exhibits a Planck distribution
   (E) $f(E)$ is independent of temperature
3. Which of the following has a different dimension as compared to all the others?
   (A) $m_e c^2$　(B) $k_B hT(\lambda m_e c)^{-1}$　(C) $\mu m_e c\lambda h^{-1}$　(D) $m_e^2 c^3 \lambda h^{-1}$　(E) $\mu k_B hT m_e^{-1} c^{-2}$
4. Consider the following four effects:
   I. Spin-orbit coupling;
   II. Relativistic correction to kinetic energy;
   III. Hyperfine structure;
   IV. Lamb shift.

   The fine structure splitting of the hydrogen atom primarily arises from which of these effects?
   (A) Only I.　(B) Only III.　(C) I and II.　(D) I, II, and III.　(E) I, II, III, and IV.
5. In quantum mechanics, which operator is directly related to time evolution?
   (A) Hamiltonian operator
   (B) Position operator
   (C) Momentum operator
   (D) Angular momentum operator
   (E) Parity operator
6. In quantum tunneling, if the width of a potential barrier is linearly increased while keeping the height constant, how does the transmission coefficient change? Assume that the particle energy is much less than the barrier height.
   (A) Remains constant.
   (B) Decreases linearly.
   (C) Increases linearly.
   (D) Decreases exponentially.
   (E) Increases exponentially.

見背面

7. Which of the following best describes Bohr's correspondence principle?
   (A) In the limit of large quantum numbers, quantum theory should approach classical theory.
   (B) Quantum mechanics must agree with Newtonian mechanics at macroscopic scales.
   (C) All quantum systems can be described by classical mechanics.
   (D) The uncertainty principle does not apply in the macroscopic world.
   (E) The phase of the wave function is proportional to the classical action.

8. In quantum mechanics, the propagator $K(x,t;x',0)$ for a free particle of mass m satisfies the time-dependent Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}K(x,t;x',0) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}K(x,t;x',0)$$

with the initial condition $\lim_{t\to 0^+} K(x,t;x',0) = \delta(x-x')$. The physical meaning of $|K(x,t;x',0)|^2$ is:
   (A) The probability of finding the particle at position x at time t.
   (B) The energy eigenvalue of the free particle.
   (C) The momentum distribution of the particle.
   (D) The probability density for a particle initially at x' to be found at x after time t.
   (E) The expectation value of position at time t.

第二大題：填充題 (fill-in-the-blank questions)

共 6 題，請依題號順序將答案寫於試卷內，無需列出計算過程，計算過程不計分。答對一題得 6 分，答錯或空白 (未答題) 皆不計分。

9 & 10. A particle with rest mass m and kinetic energy $3mc^2$ collides with an identical particle at rest. After the collision, the two particles stick together to form a composite particle. The total momentum of the system is p_{total} = __(9)__. The rest mass of this composite particle is M = __(10)__.

11 & 12. Consider a particle of mass m confined in a one-dimensional infinite square well with the potential:

$$V(x) = \begin{cases} 0, & 0 < x < a \\ \infty, & \text{otherwise.} \end{cases}$$

Use the following trial wave function: $\psi(x) = Nx(a-x)$ for $0 \le x \le a$ and $\psi(x) = 0$ otherwise. The normalization constant is N = __(11)__. The energy expectation value for this trial wave function is $\langle E \rangle$ = __(12)__.

13 & 14. Consider a two-level quantum system whose Hamiltonian operator has the following matrix representation in the basis $\{|1\rangle, |2\rangle\}$:

$$\hat{H} = \begin{pmatrix} E_0 & V \\ V & E_0 \end{pmatrix},$$

where E_0 and V are both real numbers with $V > 0$.

The energy eigenvalues of this system are __(13)__.

If the system is in the initial state $|\psi(0)\rangle = |1\rangle$ at $t = 0$, then the earliest time at which the probability of finding the system in state $|2\rangle$ reaches its maximum value is t_2 = __(14)__.