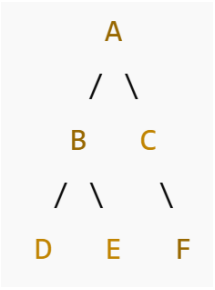
國立高雄大學 115 學年度研究所碩士班招生考試試題

科目：離散數學與資料結構　　系所：資訊工程學系　　是否使用計算機：否
考試時間：100 分鐘　　本科原始成績：100 分

資料結構:

For Questions 1 to 5, please choose the best answer to each question.

1. (5%) Given the following binary tree:



Which of the following is the correct post-order traversal?

(a) A B D E C F

(b) D E B F C A

(c) D B E A C F

(d) B D E C F A

2. (5%) In a singly linked list, deleting a node *p* after a given node *prev* requires which operation?

(a) Set p = prev->next->next

(b) Set prev->next = p->next

(c) Set prev = p->next

(d) Swap the data of p and prev

3. (5%) A graph has 10,0000 vertices but fewer than 20,000 edges. Which representation is the most memory-efficient?

(a) Adjacency matrix

(b) Edge list stored in a 2D array

(c) Adjacency list

(d) Complete graph representation

4. (5%) Which statement is true for any valid Binary Search Tree (BST)?

(a) The left subtree contains keys less than or equal to the root.

(b) The right subtree contains keys greater than or equal to the root.

(c) In-order traversal visits nodes in non-decreasing order.

(d) Pre-order traversal always produces a sorted list.

5. (5%) A hash table uses open addressing with quadratic probing and has:

- Table size: m = 20
- Number of stored keys: n = 14
- Load factor threshold for rehash: $\alpha \geq 0.70$

Inserting one more key (the 15-th key) will cause the load factor to reach:

$$\alpha = \frac{15}{20} = 0.75$$

What will the table do after the insertion attempt?
(a) The insertion succeeds; no rehash is needed
(b) The insertion succeeds; rehash is triggered after insertion
(c) The insertion is rejected; rehash occurs before the key is inserted
(d) Rehash occurs, the table size doubles to 40, and the key must be inserted into the new table

6. A Generative AI (GenAI) model processes a user's prompt and then produces a corresponding response. For example:
User prompt:
"Explain quantum mechanics in a sentence."
GenAI response:
"Quantum mechanics says that microscopic particles behave probabilistically and are changed by being measured."
(a) (10%) Propose an appropriate data structure for storing the user's prompt so that a GenAI model can interpret its content.
(b) (10%) Based on the data structure you proposed in part (a), design an algorithm that enables the GenAI model to generate the response.

7. The following pseudo code attempts to implement Huffman tree construction. You are asked to read the code and answer the questions below.
Data Structures
Each Node contains:

- symbol — character (null for internal nodes)
- freq — integer frequency
- left, right — child pointers

A **MinHeap** stores nodes ordered by frequency (freq).

Pseudo Code: Build Huffman Tree

```
BUILD-HUFFMAN-TREE(symbols, freqs):

    create an empty MinHeap H

    for i from 1 to length(symbols):
        node ← new Node
        node.symbol ← symbols[i]
        node.freq   ← freqs[i]
        node.left   ← null
        node.right  ← null
        INSERT(H, node)

    while SIZE(H) > 0:
        left  ← EXTRACT-MIN(H)
        right ← EXTRACT-MIN(H)

        parent ← new Node
        parent.symbol ← null
        parent.freq   ← left.freq + right.freq
        parent.left   ← left
        parent.right  ← right

        INSERT(H, parent)

    root ← EXTRACT-MIN(H)
    return root
```

(a) (10%) Please identify any errors in the provided code.

(b) (15%) If errors are present, please fix the code and explain why it fixes the problem.

離散數學：

8. Each of the one million car license plates labeled from 000000 to 999999 can be denoted as $x_1x_2x_3x_4x_5x_6,\ 0 \le x_i \le 9,\ 1 \le \mathrm{i} \le 6.$

(a) (3%) In how many of these car licenses are all the six digits **distinct**?

(b) (3%) In how many of these car licenses are the six digits in the relationship $x_1 < x_2 < x_3 < x_4 < x_5 < x_6$?

(c) (3%) In how many of these car licenses are the six digits in the relationship $x_1 \le x_2 \le x_3 \le x_4 \le x_5 \le x_6$?

(d) (3%) In how many of these car licenses does the digit 2 appear?
(e) (3%) In how many of these car licenses does **at least one** of the digits 0, 1, and 2 appear?
(f) (6%) In how many of these car licenses does **at least two** of the digits 0, 1, and 2 appear?

9. (9%) For a set of three distinct numbers, we can order them as $a < b < c$ where b is the middle, a is the smallest, and c is the largest. From this hint, give a combinatorial proof that

$$1 \times n + 2 \times (n-1) + 3 \times (n-2) + \cdots + (n-1) \times 2 + n \times 1 = \binom{n+2}{3},\ n \in Z^+.$$