

國立中央大學 115 學年度碩士班考試入學試題

系所： 企業管理學系 碩士班 工商管理丙組(一般生)

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科目： 微積分

*本科考試禁用計算器

Part I Multiple-Choice Questions:

The questions or incomplete statements that follow are each followed by four suggested answers or completions. Choose the response that best answers the question or completes the statement. There are 12 multiple choice questions. 3 points for each question. Please mark your answer in the multiple-choice answer card.

1. Which of the following statements about the relationship between limit, continuity and differentiability is true?

(A) If f is continuous at $x=0$, then f is derivative at $x=0$

(B) $f(x) = |x|$ is differentiable for all x

(C) If f is derivative on $[-1, 1]$, then f is continuous at $x=0$

(D) The existence of $\lim_{x \rightarrow a} f(x)$ depends on the continuity of f at $x=a$ with $a \in (b, c)$

2. Consider the following function:

$$f(x) = \begin{cases} 5x + 1, & \text{if } x < 2 \\ 13, & \text{if } x = 2 \\ x^2 + 7, & \text{if } x > 2 \end{cases}$$

Which of the following are true statements?

I. f is continuous at $x=2$;

II. $\lim_{x \rightarrow 2} f(x)$ does not exist;

III. f is derivative at $x=3$

IV. No removable points exist.

(A) I, II

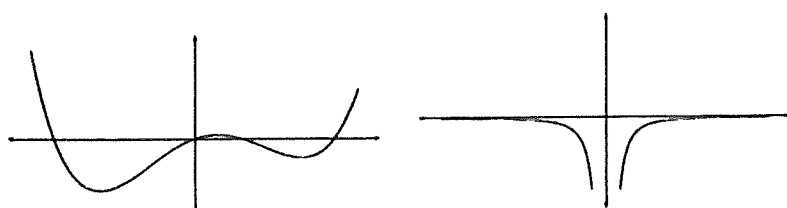
(B) I, III

(C) II, IV

(D) None

3. We wish to match each of the following graphs of $f(x)$ to the graph of its derivative $f'(x)$. Which has a correct pairing?

(A)



注意:背面有試題

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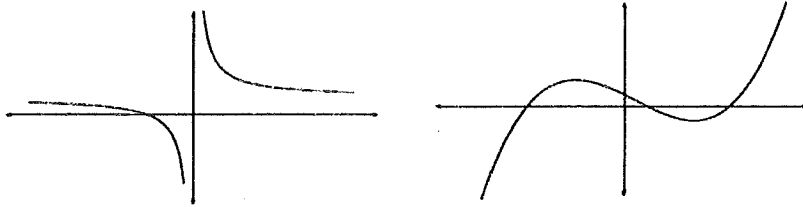
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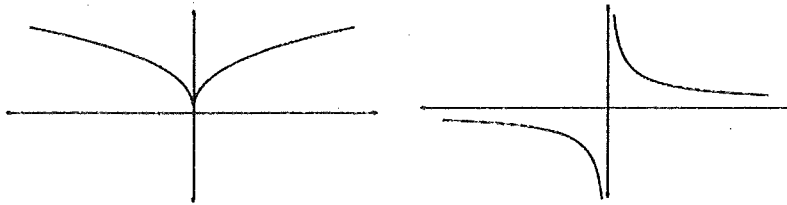
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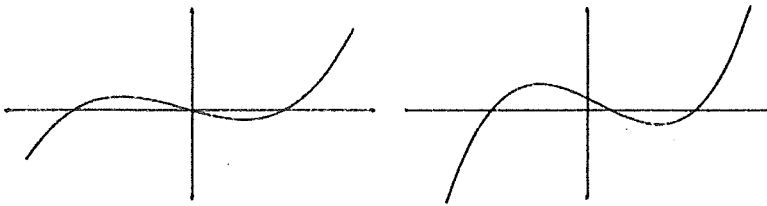
(B)



(C)



(D)



4. Which of the following statements about composite function is false?

(A) Let $f(x) = x - 5$ and $g(x) = \frac{1}{x-1}$. Then the domain of $g \circ f(x)$ is $\{x \mid x \in$

$\mathbb{R}, x \neq 1\}$

(B) Let $f(x) = x - 5$ and $g(x) = \frac{1}{x-1}$. Then the domain of $f \circ g(x)$ is $\{x \mid x \in$

$\mathbb{R}, x \neq 1\}$

(C) Let $f(x) = x - 5$ and $g(x) = \frac{1}{x-1}$. Then the rang of $g \circ f(x)$ is $\{y \mid y \in$

$\mathbb{R}, y \neq 0\}$

(D) Let $f(x) = x - 5$ and $g(x) = \frac{1}{x-1}$. Then the rang of $f \circ g(x)$ is $\{y \mid y \in$

$\mathbb{R}, y \neq -5\}$

5. Which of the following statements is always true for a function $f(x)$?

- I. If $f(x)$ is concave up on the interval (a, b) , then $f(x)$ has a local minimum on the interval (a, b)

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II. It is possible for $y=f(x)$ to have an inflection point at $(a, f(a))$ even if $f'(x)$ does not exist at $x=a$

III. It is possible for $(a, f(a))$ to be both a critical point and an inflection point of $f(x)$

(A) I and II

(B) Only III

(C) I, II, and III

(D) II and III

6. Which of the following statements about the graph of the function below is false?

$$f(x) = \frac{x^2 - 2x - 3}{x - 1}$$

(A) There are one vertical asymptotes, no horizontal asymptotes and one slant asymptote

(B) Though the function is discontinuous at $x=1$, $\lim_{x \rightarrow 1} f(x)$ still exists.

(C) There are no inflection points

(D) The range of function $f(x)$ is the real line

7. If a function f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b) , then there exists $c \in (a, b)$ such that $f(b) - f(a) = f'(c)(b - a)$ is:

(A) The Extreme Value Theorem

(B) The Intermediate Value Theorem

(C) The Mean Value Theorem

(D) Rolle's Theorem

8. Which of the following statements about the antiderivatives is true?

(A) The derivative of an antiderivative of a function is equal to the function itself

(B) Every function has an antiderivative

(C) Every differentiable function has an antiderivative

(D) None of above

9. Which of the following statements about the Reimann sum is true?

I. Riemann sums are only useful for computing definite integrals because they cannot be used to compute antiderivatives

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II. Our integration techniques only apply to definite integrals, so we can't apply them to indefinite integrals

III. The fundamental theory of calculus connects Riemann sums with antiderivatives

IV. Riemann sums always exist for any function

(A) I and II

(B) I and III

(C) II and IV

(D) III and IV

10. Which integral does not converge?

(A) $\int_4^{\infty} \frac{1}{x(\ln x)^3} dx$

(B) $\int_{-\infty}^{\infty} 2xe^{-3x^2} dx$

(C) $\int_{-1}^1 \frac{1}{x^2} dx$

(D) $\int_1^{\infty} \frac{1}{x^p} dx$ if $p > 1$

11. If $F(x) = \int_0^x \sqrt{t^3 + 1} dt$, then $F'(2) = ?$

(A) -3

(B) -2

(C) 2

(D) 3

12. Consider $\frac{x^4 + 10x^2}{(x^2 + 1)(x^2 + 9)}$. How many of the following statements are true?

I. The largest coefficient that appears when you do the partial fraction decomposition is $9/8$

II. $\frac{x^4 + 10x^2}{(x^2 + 1)(x^2 + 9)} = \frac{Ax + B}{x^2 + 1} + \frac{Cx + D}{x^2 + 9}$ for some constants $A, B, C,$ and D

III. In the $\frac{Ax + B}{x^2 + 1}$ term, $A = 0$.

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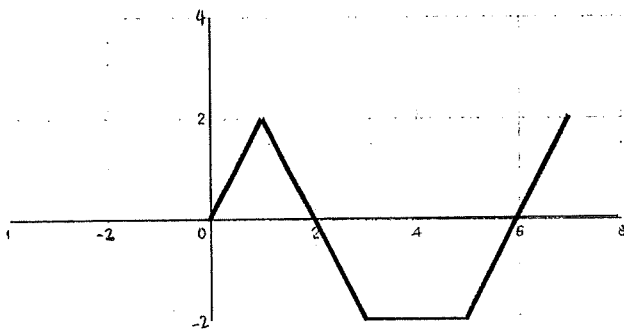
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- (A) 0
- (B) 1
- (C) 2
- (D) 3

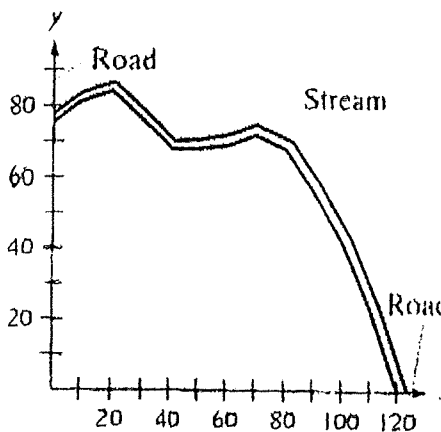
Part II Calculation problems and proof problems. Please clearly label the question and write clearly on the answer sheet. There are 8 problems and 8 points for each question.

1. Evaluate $\lim_{x \rightarrow 0} \frac{e^x + e^{-x} - 2}{1 - \cos 3x}$

2. Based on the following graph of $f(x)$, evaluate $\int_0^7 f(x) dx$



3. Use the Trapezoidal Rule to estimate the number of square meters of land in the lot, where x and y are measured in meters, as shown in the figure. The land is bounded by a stream and two straight roads. (The number of subintervals is shown in the figure)



X	Y
0	75
10	81
20	84
30	76
40	67
50	68
60	69
70	72
80	68
90	56
100	42
110	23
120	0

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4. Let f be a differentiable function, and $f\left(\frac{x-1}{x+1}\right) = x$. Find $f'(2)$.

5. Please use total derivatives to find the approximate value of $\sqrt{(2.95)^2 + (3.95)^2}$.

6. Please determine whether the following series is convergent or divergent?

$$\sum_{n=1}^{\infty} \frac{n!}{n^n}$$

7. Evaluate $\int_0^2 \int_{y^2}^4 y \cos x^2 dx dy$

8. A manufacturer produces two products, x and y . The profit function (in thousands of dollars) is:

$$P(x, y) = 10x + 12y$$

Due to limited resources, production must satisfy the constraint

$$x^2 + 2y^2 = 100,$$

where $x \geq 0$ and $y \geq 0$. Use the method of Lagrange multipliers to find the values of x and y that maximize profit? What is the maximum profit?