國立臺灣大學 115 學年度碩士班招生考試試題

Part 1: Signals and Systems

(1) Please illustrate the following terms: (6%)
(a) Gibb's phenomenon; (b) Hilbert transform

(2) Please determine (16%)

(a) $\int_{-\infty}^{\infty} \frac{\sin(2\pi t)}{t} dt$,

(b) $\int_{-\infty}^{\infty} \frac{\sin^2(2\pi t)}{t^2} dt$

(c) $\int_{-\infty}^{\infty} \sin(30\pi\tau)\cos(6\pi(t-\tau))\, d\tau$,

(d) $\int_{-\infty}^{\infty} \frac{\sin(30\pi\tau)}{\tau}\cos(6\pi(t-\tau))\, d\tau$.

(3) Please determine the inverse Z transforms of the following functions: (12%)
(It is OK to show only the causal solution).

(a) $\frac{1+z^{-1}}{4-3z^{-1}-z^{-2}}$

(b) $\log(1+2z^{-2})$

(4) Suppose that $X(j\omega)$ is the continuous Fourier transform of $x(t)$ (16%)
(a) What is the Fourier transform of
$\mathcal{Re}\{x(3t+6)\}$ where $\mathcal{Re}\{\ \}$ means taking the real part?
(b) What is the Fourier transform of $X(jt)$?
(c) Suppose that the bandwidth of $x(t)$ is W. Please estimate the bandwidth of $|x(t+1)|^2$.
(d) Continue in (c), how do we sample $|x(t+1)|^2$ without aliasing effect?

Part 2: Principles of Communication
(Please simplify your answers as much as possible and write down detailed steps)

(5) In a fading channel the received signal is $y = hx + w$, where x is a quadrature-amplitude modulation (QAM) signal and both the fading coefficient h and noise w are zero-mean complex-Gaussian distributed with variance $E[|h|^2] = E[|w|^2] = 1$.
(a) Please find the variance of the real part of h. (6%)
(b) Assume that

$$x \in \{\sqrt{P}(m_1 + jm_2) | m_1, m_2 = -1,0,1,2\},$$

also the point $x = \sqrt{P}(1-j)$ is sent. The receiver knows the realization of h. Please find the set of QAM constellation points having the largest pairwise error probability to x under the maximum likelihood (ML) rule. (6%)

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(c) Please use union-bound and Q-function to find the upper-bound of the symbol error probability of ML demodulator given realization $h = h'$. You can simplify the answer assuming P is high. (8%)

(d) Using the Q-function upper-bound $Q(t) \leq (1/2)e^{-t^2/2}$, based on your simplified answer in (c), please find the upper-bound of the symbol error probability when $|h|^2$ is Chi-squared distributed with probability density function (PDF) $f(t) = e^{-t}, t \geq 0$. (12%)

(6) Consider coding in a binary finite field. A 1-bit coded packet $[\boldsymbol{w}_1 \boldsymbol{w}_2]\boldsymbol{c}_t$ is sent at discrete time index t, where message vectors $\boldsymbol{w}_1$ and $\boldsymbol{w}_2$ are 4×1 and 4×1 binary row vectors respectively while $\boldsymbol{c}_t$ is binary 1×8 coding vector. Let spaces Ω_1 be span$\{\boldsymbol{\delta}_j : j = 1 \ldots 4\}$ and Ω_2 be span$\{\boldsymbol{\delta}_j : j = 1 \ldots 8\}$ where $\boldsymbol{\delta}_j$ is 8×1 binary elementary basis with the j-th element being 1. In noiseless channel, the receiver has knowledge space S(t) being span$\{\mathrm{c}_\tau : \tau = 1 \ldots t\}$

(a) Let
$\boldsymbol{c}_1 = [0,1,0,0,0,1,0,0]^T$ and $\boldsymbol{c}_2 = [0,1,0,0,0,0,0,0]^T$ where T is the transpose.
Find the rank y^t = rank(S(t) $\cap\, \Omega_2$) at t =1 and 2 where $\cap$ is set intersection. (8%)

(b) Continue from (a), find Y^t = rank(S(t)$\oplus\Omega_1$) − rank(Ω_1) at t =1 and 2, where the linear sum space S(t)$\oplus\Omega_1$ is the span of all vectors belong to set union S(t) $\cup\, \Omega_1$. (10%)