*各題答案須填寫於答案卡上，未依規定作答者不予計分。
*每題有一個或多個正確選項，須完全作答正確（未誤選亦未漏選）方可獲得該題滿分。
*每題若未作答，或作答錯誤（包含應選而未選，或不應選而選），該題一律以零分計算。

1. (5%) The isometric knee-extension contraction in a standing posture is modeled as a one-dimensional system consisting of two point masses connected in series by two linear springs representing tendons. From bottom to top, the system comprises: the patellar tendon (spring constant k_1), the patella (mass m_1), the quadriceps tendon (spring constant k_2), and the quadriceps muscle (mass m_2). The lower end of the patellar tendon is attached to the tibia, which is assumed to remain stationary during the contraction. All motion is restricted to the vertical z-direction, with positive z defined upward. Let $z_1(t)$ and $z_2(t)$ denote the vertical displacement of mass m_1 and m_2, respectively, from their equilibrium positions. Assume the tendon mass is negligible, $k_1= 6$, $k_2 = 4$, $m_1 = 1$, $m_2 = 1$, $z_1(0) = 0$, $\dot{z}_1(0) = 0$, $z_2(0) = 0$, $\dot{z}_2\ (0) = 1$. Which of the following statements are true?

(A) $z_1(t) = \frac{\sqrt{2}}{10}\sin\sqrt{2}t - \frac{\sqrt{3}}{5}\sin\sqrt{3}t$

(B) $z_1(t) = \frac{\sqrt{2}}{5}\sin\sqrt{2}t - \frac{\sqrt{3}}{15}\sin 2\sqrt{3}t$

(C) $z_2(t) = \frac{\sqrt{2}}{5}\sin 2\sqrt{2}t + \frac{\sqrt{3}}{15}\sin\sqrt{3}t$

(D) $z_2(t) = \frac{2\sqrt{2}}{5}\sin\sqrt{2}t + \frac{\sqrt{3}}{30}\sin 2\sqrt{3}t$

(E) None of above.

2. (5%) Consider a cell migrating on a deformable substrate lying in the x-y plane. The cell is initially located at the origin (0, 0). There are three markers, a_1, a_2, and a_3, embedded in the substrate at initial positions a_1 = (0,0), a_2 = (0,1), and a_3 = (1,0). Following cell-induced deformation, the marker positions become a_1 = (0,0), a_2 = (0.5, 2.5), and a_3 = (2.5, 0.5). Let F denote the deformation gradient. The principal directions and magnitudes of the substrate deformation can be determined by solving the following eigenvalue problem:

$$e = \frac{1}{2}(F^TF - I) = \begin{bmatrix} 2.75 & 1.25 \\ 1.25 & 2.75 \end{bmatrix}, F = \begin{bmatrix} 2.5 & 0.5 \\ 0.5 & 2.5 \end{bmatrix}$$

Which of the following statements are true?

(A) The maximum principal deformation magnitude is 4.
(B) The maximum principal deformation magnitude is 1.5.
(C) The eigenvector corresponding to the direction of maximum principal deformation is proportional to [1, 1].
(D) The eigenvector corresponding to the direction of maximum principal deformation is proportional to [1, -1].
(E) None of above.

3. (5%) A scientist investigates how M2 macrophages induce the conversion of adjacent M1 macrophages via cytokine-mediated crosstalk. The system is modeled as a spherical aggregate containing 1,000 M1 macrophages. On day 0, a single M1 macrophage is converted into an M2 macrophage. Assume that the conversion rate is proportional to both the number of already converted macrophages and the number of remaining unconverted macrophages. If approximately 120 macrophages have been converted by day 5,

見背面

which of the following is the closest to the earliest day on which nearly all macrophages in the aggregate are converted.

(A) Day 7
(B) Day 11
(C) Day 17
(D) Day 24
(E) None of above.

4. (5%) Given the initial value problem $y' = x^2 y^{1/3}$, $y(0) = 1$, which of the following statements are true?
(A) The differential equation admits infinitely many solutions.
(B) $y < x$ for all $x \in (0,1)$.
(C) $y \gg x$ for $x > 1$.
(D) $y \simeq 1$ as $x \to -\infty$.
(E) None of above.

5. (5%) The mechanical behavior of human tissue can be modeled using the Kevin-Voigt model, which consists of a linear spring with elastic modulus E connected in parallel with a dashpot with viscosity of η. The governing constitutive equation is

$$\sigma(t) = E\varepsilon(t) + \eta \frac{d\varepsilon(t)}{dt}, \qquad \varepsilon(0) = 0,$$

where σ, ε represents the applied stress and the resulting strain, respectively. Assume a constant stress σ_1 is applied for $0 \le t \le 10$, after which the stress is removed, which of the following statement are true?
(A) ε remains constant for $0 \le t \le 10$.
(B) $\varepsilon = 0$ for $t > 10$.
(C) ε reaches its maximum at $t = 0$ and then decreases gradually.
(D) ε reaches its maximum $t = 10$ and then decreases gradually.
(E) None of above.

6. (5%) A new drug with a molecular weight of 199.1 g mol^{-1} is delivered using a square transdermal patch of dimensions 51 mm × 51 mm and thickness 2 mm. The patch initially contains 700 mg of the drug. The patch is applied to a region of skin with an average thickness of 650 nm. Owing to a dense capillary network in this region, any drug that crosses the skin is immediately removed by the circulatory system, maintaining a negligible drug concentration at the inner skin boundary. Let x denote the coordinate normal to the skin surface, with $x = 0$ at the outmost surface of the skin. Assume the drug flux does not change as the concentration of drug in the patch decreases; the drug concentration varies only along the x-direction; and the diffusion constant of the drug molecule is 6.9×10^{-9} cm^2 s^{-1}. Which of the following statements are true?
(A) At steady state, the drug concentration $c(x)$ changes linearly with x.
(B) The drug flux across the skin is about 7×10^{-8} mol cm^{-2} s^{-1}.
(C) About 10% drug remains in the patch after 30 minutes of application.
(D) It takes approximately 12 hours to completely deliver the drug from the patch.
(E) None of above.

接次頁

7. (5%) Consider the 2nd order differential equation $y'' = 2x(y')^2$, which of the following statements are true?
(A) $y' \to 0$ as $x \to \infty$.
(B) $y'(a) = y'(-a)$ for all $a \in (-1,\ 1)$.
(C) $y(1) = y(-1)$.
(D) $y \to \infty$ as $x \to \infty$.
(E) None of above.

8. (5%) Which of the following plane autonomous systems have infinite critical points?
(A) $\dot{x} = -xy + 3y,\ \dot{y} = x + 3y$.
(B) $\dot{x} = -x^2 + 4y^2,\ \dot{y} = x^2 - 4y^2$.
(C) $\dot{x} = x^2 - xy + 6,\ \dot{y} = x^2 + x$.
(D) $\dot{x} = x^2 - xy + 12x,\ \dot{y} = y^2 - x + 4y$.
(E) None of above.

9. (5%) A microfluidic channel is used to study T-cell chemotaxis. A T cell is initially located at the midpoint of a 100-μm-long channel and is approximated as a sphere of diameter 10 μm. The concentration of a chemoattractant X is maintained at steady state, with c =1 μM at the front end of the channel and c = 0 at the back end. Assume one-dimensional steady-state diffusion. All experiments are conducted at T = 300 K. The diffusion constant of X in water is 2.2×10^{-6} $m^2\,s^{-1}$. What is the concentration difference experienced between the front and back of the cell?
(A) 0.01 μM.
(B) 0.005 μM.
(C) 0.1 μM.
(D) 0.23 μM.
(E) None of above.

10. (5%) Given the linear system $\dot{x} = 2x + 8y,\ \dot{y} = -x - 2y,\ x(0) = 2,\ y(0) = 0$. Which of the following statements are true?
(A) The solution $x(t)$ is periodic.
(B) $x(t) = 2e^t \cos t$.
(C) $y(t) = -\cos 2t + \cos t$.
(D) The system possesses infinitely many solutions for $x(t)$ and $y(t)$.
(E) None of above.

11. (7%) Let P be an $m \times m$ matrix, Q be an $n \times n$ matrix, A be an $m \times n$ matrix, and B be an $n \times p$ matrix. Which of the following statements are true?
(A) Null $A \subseteq$ Null PA.
(B) If Q is invertible, then Null $AQ =$ Null A.
(C) Col $AB \subseteq$ Col A.
(D) Null $AB =$ Null B.
(E) The rank of AB is less than or equal to the rank of A.

見背面

12. (7%)

$$A=\begin{bmatrix}1&0&1&2\\0&1&1&1\end{bmatrix},\quad S=\left\{\begin{bmatrix}-3\\-2\\1\\1\end{bmatrix},\begin{bmatrix}-2\\-1\\0\\1\end{bmatrix},\begin{bmatrix}-3\\-2\\1\\2\end{bmatrix}\right\}.$$ Which of the following statements are true?

(A) Let T_A be the linear transformation induced by A. T_A is one-to-one.
(B) T_A is onto.
(C) The dimension of the range of $T_{A^T} = 2$.
(D) Null A = Span S.
(E) Let $\mathcal{B}_1$ be any basis for the range of T_{A^T} and $\mathcal{B}_2$ be any basis for Span S. Then $\mathcal{B}_1 \cup \mathcal{B}_2$ constitutes a basis for $\mathcal{R}^4$.

13. (7%) For every 5×5 matrix A, if Dim (Col A^T) = 3, then the multiplicity of the eigenvalue $\lambda = 0$ of A
(A) must be 2.
(B) must be 3.
(C) can be either 0, 1, or 2.
(D) can be either 3, 4, or 5.
(E) can be either 2, 3, 4, or 5.

14. (7%) Which of the following subsets are subspaces?
(A) $S_A = \{(u_1, u_2) \in R^2 \mid 2u_1^2 + 3u_2^2 \leq 12\}$.
(B) $S_B = \{x \in \mathcal{R}^n \mid Ax = 0$ for all rank-1 real matrices A of compatible size$\}$.
(C) $S_C = \{x \in \mathcal{R}^n \mid x^T Ax = 0$ for all real symmetric matrices $A\}$.
(D) $S_D = \{x \in \mathcal{R}^n \mid Ax = b\}$, where A is a fixed real matrix and b is a fixed nonzero vector.
(E) S_E = Span U ∪ Span V, where $U, V \subset \mathcal{R}^n$ are linearly independent sets.

15. (7%) Which of the following statements are true?
(A) Let P be an orthogonal projection matrix. Then Null P = Col $(I - P)$.
(B) Let C be a real matrix whose columns form a linearly independent set of vectors in R^n, and define $P = C(C^TC)^{-1}C^T$. Then Col P = Col C and Null $P = (\text{Col } C)^\perp$.
(C) Let A be a real matrix. If there exists an orthogonal matrix Q such that Q^TAQ is diagonal, then $A = A^T$.
(D) Let A be a real symmetric matrix. If $v_1, \ldots, v_k$ are linearly independent eigenvectors of A, then $v_i \perp v_j$ for all $i \neq j$.
(E) Let A be a real matrix that is diagonalizable and has only real eigenvalues. Then A is similar to a symmetric matrix.

16. (7%) Consider the vector space P of all real polynomials in x.

For $p(x) = p_0 + p_1x + p_2x^2 + \ldots + p_{n_p}x^{n_p}$

$q(x) = q_0 + q_1x + q_2x^2 + \ldots + q_{n_q}x^{n_q}$

接次頁

Determine which of the following define an inner product on P?

(A) $\langle p,q\rangle = \int_0^1 p(x)q(x)dx.$

(B) $\langle p,q\rangle = \int_0^1 p'(x)q'(x)dx$

(C) $\langle p,q\rangle = \sum_{n=0}^{\infty} p_k\, q_k$.

(D) $\langle p,q\rangle = \sum_{n=0}^{\infty}(-1)^k p_k\, q_k$

(E) $\langle p,q\rangle = \sum_{n=0}^{\infty}(k+1)^k p_k\, q_k$.

17. (8%) Let $T: P_2 \to P_2$ be defined by

$$T(p(x)) = p(0) + (p(0)+p'(0))x + (p'(0)+p(1))x^2,$$

where

$$p'(0) = \left.\frac{dp(x)}{dx}\right|_{x=0}.$$ Which of the following statements are true?

(A) T is a linear transformation.
(B) T is an isomorphism.
(C) The characteristic polynomial of T has a repeated root.
(D) T is diagonalizable.
(E) The dimension of the eigenspace corresponding to the eigenvalue $\lambda = 1$ is equal to 2.

試題隨卷繳回