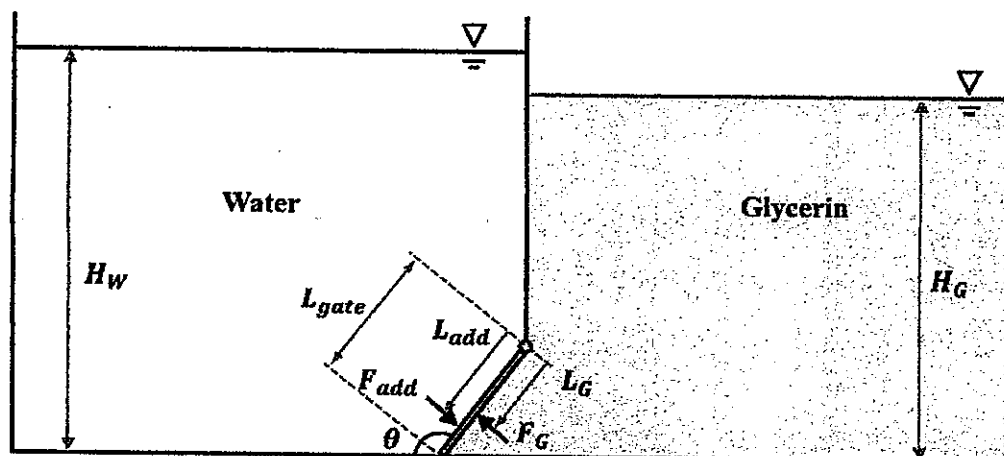


Please write down all the assumptions and steps to solve the problems. The answer could be written in Chinese or English. The units need to be indicated in the answers.

1. There is a gate that separates the liquid on the two sides of a rectangular tank. The liquid on the left-hand side is water. The liquid on the right-hand side is glycerin. The depth of the glycerin ( $H_G$ ) is 3 m. The full length of the gate ( $L_{gate}$ ) is 1.5 m. The angle between the gate and the tank bottom ( $\theta$ ) is  $120^\circ$ . The width of the tank is 3 m. The gravitational acceleration ( $g$ ) is  $9.8 \text{ m/s}^2$ . The properties of the two liquids are listed in the table below. Please answer the following questions, ignoring the weight of the gate.
  - a. What is the total force ( $F_G$ ) exerted on the gate by glycerin? Please also indicate the center of pressure ( $L_G$ ). (10%)
  - b. How much water needs to be filled into the tank to keep the gate from rotating? Please answer the question in terms of water depth ( $H_w$ ). (5%)
  - c. If the water depth is  $H_w = 2 \text{ m}$ , what is the minimum force we need to add ( $F_{add}$ ) to keep the gate from rotating? Please also indicate the location of the force ( $L_{add}$ ). (10%)

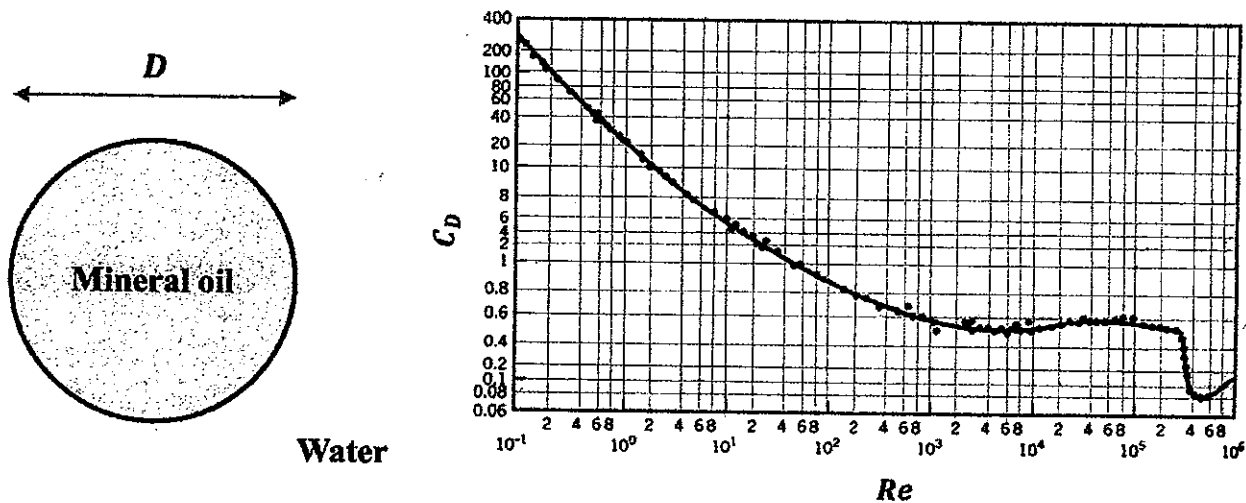


Properties	Water	Glycerin
Molar mass [ $\text{g} \cdot \text{mol}^{-1}$ ]	18.015	92.094
Dynamic Viscosity [ $\text{Pa} \cdot \text{s}$ ]	0.00089	1.412
Density [ $\text{kg/m}^3$ ]	997	1260
Refractive index	1.333	1.4746

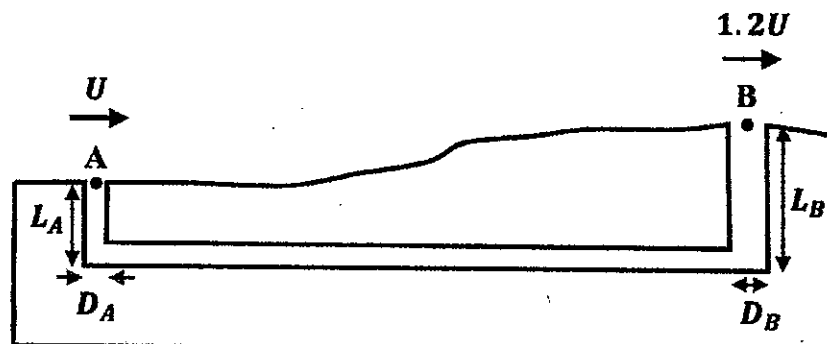
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2. There is a hollow sphere submerged in the water. The sphere is filled with mineral oil. The density of the mineral oil is  $850 \text{ kg/m}^3$ . The drag force exerted by the water ( $F_D$ ) is related to the velocity of the sphere ( $U$ ) such that  $F_D = 0.5\rho AC_d U^2$ . Here,  $\rho$  is the liquid density,  $A$  is the cross-sectional area of the sphere, and  $C_D$  is the drag coefficient, which depends on the Reynolds number  $Re = UD/\nu$ .  $D$  is the sphere diameter.  $\nu = 8.9 \times 10^{-6} \text{ m}^2/\text{s}$  is the kinematic viscosity of water. The relationship between  $C_D$  and  $Re$  is shown in the figure below. Please answer the following questions, ignore the thickness and weight of the sphere shell.

- The diameter of the sphere is 10 cm. What is the buoyancy force exerted by the water? Hint: the volume of a sphere is  $V_{ols} = \frac{1}{6}\pi D^3$ . (5%)
- What is the gravitational force acting on the sphere? (5%)
- In which direction does the sphere move (up or down)? Please justify your answer. (5%)
- Consider the gravitational force, buoyancy force, and drag force exerted by the water. Please estimate the terminal velocity of the sphere  $U$ . Please also indicate the  $C_D$  you estimated. (10%)



3. There is a wind blowing over a venting system. Air density ( $\rho_{air}$ ) is  $1.2 \text{ kg/m}^3$ .  $L_A$  is 1.5 m, and  $L_B$  is 3 m. The diameter of the pipe at Point A is  $D_A = 0.3 \text{ m}$ . The diameter of the pipe at Point B is  $D_B = 1 \text{ m}$ .  $U = 1.5 \text{ m/s}$ .
- Calculate the pressure difference between Point A and Point B using the Bernoulli equation. (5%)
  - The flow rate in the venting system is  $Q = 0.01 \text{ m}^3/\text{s}$ ; please calculate the cross-sectional averaged flow velocity at Point A ( $V_A$ ) and Point B ( $V_B$ ). (5%)
  - Please indicate the direction of the flow in the pipe, and justify your answer. (5%)
  - Calculate the head loss due to the pipe friction using the energy equation; you may assume the kinetic energy correction factor is 1. (10%)



4. There is a tank filled with water. The tank area ( $A_{\text{Tank}}$ ) is  $40 \text{ m}^2$ . The diameter of the two inlets ( $D_A$  and  $D_B$ ) is  $0.2 \text{ m}$  and  $0.5 \text{ m}$ , respectively. The flow velocity in Inlet A ( $U_A$ ) and Inlet B ( $U_B$ ) is  $0.12 \text{ m/s}$  and  $0.05 \text{ m/s}$ , respectively. The hole is  $0.5 \text{ m}$  above the tank bottom ( $H_H = 0.5 \text{ m}$ ). The hole diameter ( $D_H$ ) is  $0.15 \text{ m}$ .
- Find the velocity at the hole ( $U_H$ ) as a function of water depth ( $H_w$ ) for  $H_w \gg H_H$  using the Bernoulli equation, assuming that the flows of two inlets and outflow from the hole have negligible impact on the water surface elevation in the tank. (5%)
  - The current water depth ( $H_w$ ) is 4 meters. The water jet from Point A is deflected  $60^\circ$  by a vane ( $\theta = 60^\circ$ ). Assume jet diameter is constant and neglect gravitational effect; please find the force exerted by the jet on the vane in horizontal ( $F_x$ ) and vertical ( $F_y$ ) directions. (10%)
  - Determine if the tank is filling or emptying. Please justify your answer using the concept of control volume. (5%)
  - How fast is the water surface elevation rising or falling in the tank? (5%)

