考試科目	統計學	系所別	風險管理與保險學系精算科學組	考試時間	2 月 6 日 (五) 第四節

1. (20 pts) Please explain the following items.

   (a) (5 pts) Moment-Generating Function

   (b) (5 pts) Type I Error

   (c) (10 pts) Describe the desirable statistical properties of an estimator.

2. (10 pts) A portfolio consists of 100 individual risks divided equally into two categories, X and Y, with 50 risks in each. The losses from risks in category X have an average of 10 and a standard deviation of 5. The portfolio as a whole has an average loss of 20 and a standard deviation of 15. What is the standard deviation of losses for risks in category Y?

   (a) Less than 9

   (b) At least 9, but less than 13

   (c) At least 13, but less than 17

   (d) At least 17, but less than 21

   (e) At least 21

3. (10 pts) A researcher studies the relationship between sales performance of a life Insurance product and three independent variables: years of experience, hours of training, age. A multiple regression model is fitted using data from 1,200 employees. According to the assumptions of multiple regression, which variable is required to have a Normal distribution with constant variance ?

   (a) Years of experience.

   (b) Hours of training.

   (c) Age.

   (d) Sales performance of a life Insurance product

   (e) All Variables.

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4. (15 pts) Let X_1, X_2, and X_3 be independent random variables from a Poisson distribution with mean 1, i.e., $X_i \sim \text{Poisson}(1)$ for $i = 1, 2, 3$. Define

$$Y_1 = X_1 + X_3, \quad Y_2 = X_2 + X_3,$$

and indicator variables

$$Z_i = 1_{\{Y_i = 0\}}, \quad i = 1, 2.$$

Compute the correlation $\text{Corr}(Z_1, Z_2)$.

5. (15 pts) Let X and Y be continuous random variables with cumulative distribution functions $F_X(x)$ and $F_Y(y)$, respectively. Derive the conditional distribution of Y given that $X - Y = 0$.

6. (30 pts) Let $X_1, \ldots, X_n$ be a random sample from a normal distribution with mean μ and variance σ^2, where σ^2 is an unknown constant. Consider the hypotheses

$$H_0: \ \mu \geq \mu_0 \qquad \text{versus} \qquad H_A: \ \mu < \mu_0.$$

(a) (8 pts) Derive the likelihood ratio test of size α, $0 < \alpha < 1$.

(b) (7 pts) Express the corresponding p-value of the likelihood ratio test based on the observed values $x_1, \ldots, x_n$ of a random sample.

(c) (7 pts) Compute the power of the test at $\mu = \mu_1$ with $\mu_1 < \mu_0$.

(d) (8 pts) Construct the uniformly most accurate $(1 - \alpha)$ confidence interval for μ.