考試科目	微積分	系所別	應用經濟與金融英語碩士學位學程	考試時間	2月6日(五)第2節

Instructions

Answer all problems. Provide full reasoning and clear justification for each answer.

Problems: each 10 points

1. (Continuity and differentiability) Define

$$f(x) = \begin{cases} \dfrac{\sin x}{x}, & x \neq 0, \\ a, & x = 0. \end{cases}$$

   (a) Determine the value of a such that f is continuous at $x = 0$.
   (b) Determine whether $f'(0)$ exists, and if so, compute it.

2. (Two-variable limit) Evaluate, and justify existence or nonexistence of,

$$\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^2 + y^2}.$$

3. (Series convergence) Determine whether the series

$$\sum_{n=1}^{\infty} \frac{\ln n}{n}$$

converges or diverges. Give reasons.

4. (Limit proof) Calculate the limit

$$\lim_{x\to 0} \frac{e^x - 1 - x}{x^2}.$$

5. (Substitution integral) Evaluate the integral

$$\int_0^1 \frac{\ln(1+x)}{1+x^2}\, dx.$$

6. (Double integral) Compute

$$\iint_D (x+y)\, dA, \qquad D = \{(x,y) \mid x \geq 0,\ y \geq 0,\ x+y \leq 1\}.$$

7. (Uniform convergence and differentiation) Let

$$f_n(x) = \frac{x}{1+nx^2}, \qquad x \in R.$$

   (a) Find the pointwise limit $f(x)$.
   (b) Determine whether $\{f_n\}$ converges uniformly.
   (c) Discuss whether limit and differentiation may be interchanged.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	線性代數	系所別	應用數學系(含資料科學碩士班)	考試時間	2月26日(三)第2節

8. (Parameter-dependent integral) For $a > -1$, define

$$I(a) = \int_0^1 x^a \ln x \, dx.$$

(a) Compute $I(a)$.
(b) Evaluate $\lim_{a \to 0} I(a)$.

9. (Changing order of integration) Evaluate

$$\int_0^1 \int_y^1 \frac{1}{1+x^2} \, dx \, dy$$

by reversing the order of integration.

10. (Interchanging summation and integration) Consider

$$\int_0^1 \left(\sum_{n=1}^{\infty} x^{n^2} \right) dx.$$

(a) Justify whether one may interchange sum and integral.
(b) Evaluate the resulting expression.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。