國立政治大學 115 學年度 碩士班暨碩士在職專班 招生考試試題

考 試 科 目	數理統計學	系 所 別	統計學系	考 試 時 間	2月6日(五) 第 二 節

1. (10%) Let X be a random variable and $p \in (0,\infty)$. Assume the p-th moment of X exists. Prove that

$$E|X|^p = \int_0^\infty p t^{p-1} Pr(|X| > t)\, dt .$$

2. (10%) Let $\mu(z) = E(Y \mid Z = z)$. Show that

$$\frac{Var(\mu(Z))}{Var(Y)} = \text{Corr}^2(Y, \mu(Z)) = \sup_{g \in L^2(Z)} \text{Corr}^2(Y, g(Z))$$

where the supremum is taken over all square-integrable functions of Z.

3. Let $\{\tilde{X}_1, \ldots, \tilde{X}_N\}$ and $\{(X_1, Y_1), \ldots, (X_n, Y_n)\}$ be independent sets of random samples from the same population.

   (a) (10%) Given a prediction function f, show the estimator

$$\hat{\theta} = \frac{1}{N}\sum_{j=1}^{N} f(\tilde{X}_j) + \frac{1}{n}\sum_{i=1}^{n}(Y_i - f(X_i))$$

   is unbiased for $\theta = E(Y)$.

   (b) (10%) Find the condition on the correlation between Y and $f(X)$ to make $\hat{\theta}$ have smaller MSE than $\bar{Y}_n = n^{-1}\sum_{i=1}^{n} Y_i$ in terms of the standard deviations of Y and $f(X)$.

4. Let $X_1, \ldots, X_n$ be i.i.d $N(\mu, \sigma^2)$ random variables with unknown $\mu \in R$ and known $\sigma^2 > 0$.

   (a) (15%) Find the UMVUE of $\theta = e^{t\mu}$ with a fixed $t \neq 0$.

   (b) (15%) Determine whether the variance of the UMVUE in (a) attains the CRLB.

5. Let X_1 and X_2 be independent Exponential random variables, with density

$$f(x;\theta_i) = \frac{1}{\theta_i} e^{-x/\theta_i}, x > 0, i = 1,2$$

   Define $\gamma = \theta_1/\theta_2$. Find a LR test of size α for testing

   (a) (15%) $H_0: \gamma = 1$ versus $H_1: \gamma \neq 1$

   (b) (15%) $H_0: \gamma \leq 1$ versus $H_1: \gamma > 1$

備 註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。