考試科目	統計學A	系所別	金融學系 金融管理組	考試時間	2月6日(五)第四節

1. (20 points) Let the random variable N be the number of defaults in a bond portfolio for a given fiscal year. Assume that $\mathbb{P}(N = n + 1) = 0.2 \cdot \mathbb{P}(N = n)$.
   (a) (10 points) Find the probability of no default, $\mathbb{P}(N = 0)$.
   (b) (10 points) Find the probability of more than one default, $\mathbb{P}(N > 1)$.

2. (20 points) There are 5 bonds in a portfolio. At the end of a specific time period, each bond will have a value of \$100 with probability 0.5 and $\$(100 - K)$ with probability 0.5. K is the loss that a bond would suffer if there is a default. The bond payoffs and the default events are independent. Find the *maximum* value of K such that the probability of portfolio value dropping to below \$450 is less than 10%.

3. (10 points) The two independent random variables x and y have Uniform$(0, 1)$ distribution. Let $z = \max(x, y)$. Find the probability density function of z.

4. (35 points) Suppose we specify a linear regression model,

$$y = \beta_0 + \beta_1 x + \beta_2 z + \varepsilon,$$

   and obtain the estimated parameters $\hat{\beta}_0$, $\hat{\beta}_1$, and $\hat{\beta}_2$ with ordinary least squares (OLS).
   (a) (10 points) Write down the relevant assumptions so that $\hat{\beta}_1$, and $\hat{\beta}_2$ are *unbiased* estimators. No points will be given if any irrelevant assumptions are also provided.
   (b) (5 points) Explain why $\hat{\beta}_0$ will also be biased if $\hat{\beta}_1$ is biased.
   (c) (20 points) Explain the procedure to test the hypothesis $H_o : \beta_1 = 0$ and $\beta_2 = 1$. Please elaborate on how to compute the test statistic and what the critical value is.

5. (15 points) Let H_o and H_a denote the usual null and alternative hypotheses, respectively. T is the test statistic under the null hypothesis. We reject H_o if the calculated test statistic $\widehat{T}$ is greater than the corresponding critical value t_c. Based on these notations, define
   (a) (5 points) Type I error,
   (b) (5 points) Type II error,
   (c) (5 points) and p-value.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。