國立政治大學 115 學年度 碩士班暨碩士在職專班 招生考試試題

考 試 科 目	線性代數	系 所 別	應用數學系	考 試 時 間	2月6日(五) 第四節

1. (10%) Show that if $M_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, M_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, and\ M_3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then the span of $\{M_1, M_2, M_3\}$ is the set of all symmetric 2×2 matrices.
2. (10%) Let W_1 and W_2 be subspaces of a finite-dimensional vector space V. Determine necessary and sufficient conditions on W_1 and W_2 so that $\dim(W_1 \cap W_2) = \dim(W_1)$.
3. (10%) For any angle θ, let $T_\theta(a_1, a_2)$ be the vector obtained by rotating (a_1, a_2) counterclockwise by θ if $(a_1, a_2) \neq (0,0)$, and $T_\theta(0,0) = (0,0)$. Show that $T_\theta : R^2 \to R^2$ is a linear transformation.
4. (10%) Let W be a subspace of a vector space V and $T: V \to V$ be a linear transformation. Suppose that W is T-invariant. Prove that $N(T_W) = N(T) \cap W$ and $R(T_W) = T(W)$.
5. (10%) Let V and W be vector spaces, and let T and U be nonzero linear transformations from V into W. If $R(T) \cap R(U) = \{0\}$, prove that $\{T, U\}$ is a linear independent subspace of $L(V, W)$.
6. (10%) In R^2, let L be the line $y = mx$, where $m \neq 0$. Find an expression for $T(x, y)$, where T is the projection on L along the line perpendicular to L.
7. (10%) Let A be an $m \times n$ matrix. Prove that if c is any nonzero scalar, then $\mathrm{rank}(cA) = \mathrm{rank}(A)$.
8. (10%) Suppose that the sequence $f_n, n = 1,2,3, \ldots$ satisfies $f_n = 2f_{n-1} + f_{n-2}$, $f_1 = 1$, $f_2 = 1$. Find the general solution for f_n with $n \geq 3$.
9. (10%) Suppose that $A \in M_{n \times n}(F)$ has two distinct eigenvalues, μ_1 and μ_2, and that $\dim(E_{\mu_1}) = n - 1$. Prove that A is diagonalizable.
10. (10%) Let T be an invertible linear operator on a finite-dimensional vector space V. Prove that if T is diagonalizable, then T^{-1} is diagonalizable.

備 註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。