



本試題共 7 題，每題得分如各題中所示，共計 100 分，請依題號作答並將答案寫在答案卷上，違者不予計分。

1. (15%) Please solve for $y = y(x)$.
 - (a) (05%) $xy' - 3y = 0$
 - (b) (05%) $y'' - 4y' + 4y = 0$
 - (c) (05%) $y'' + \frac{1}{x}y' - \frac{1}{x^2}y = 0$
2. (15%) Find the integration factor and solution of the ODE equation
 $(x^2 - 4y)dx - xdy = 0$
3. (10%) Solve ODE solution of $y'' - 4y' + 3y = \sin 2x$
4. (10%) Laplace equation:
 - (a) (05%) If $f(t) = \cos(2t + \theta_0)$, $t \geq 0$, please find $L[f(t)]$
 - (b) (05%) $F(S) = \frac{1}{(S+1)(S+2)}$, please find $L^{-1}[F(S)]$.
5. (15%) A transform $T(\mathbf{x}) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, where $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbb{R}^2$ and $\cdot$ denotes the inner product.
 - (a) (10%) Prove $T(\mathbf{x})$ is linear.
 - (b) (05%) If $T(\mathbf{x}) = \mathbf{Ax}$, find the matrix $\mathbf{A}$.
6. (15%) Let $W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ in } \mathbb{R}^3 : x + 2y - z = 0 \right\}$.
 - (a) (10%) Prove that W is a subspace of $\mathbb{R}^3$.
 - (b) (05%) Find the orthogonal complement of W , denoted as $W^\perp$.
7. (20%) A linear transformation $T(\mathbf{x}) = \mathbf{Ax}$, where

$$\mathbf{A} = \begin{bmatrix} -1 & -1 & 1 \\ 0 & -2 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$
 - (a) (08%) Find the eigenvalues of $\mathbf{A}$.
 - (b) (08%) Suppose $V = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ forms the eigenbasis for $\mathbf{A}$, find V .
 - (c) (04%) Find the transformation matrix of $T(\mathbf{x})$ when using V from (b) as the basis.