國立中山大學114學年度
碩士班考試入學招生考試試題

科目名稱：離散數學與演算法【資工系資安碩班碩士班】

—作答注意事項—

考試時間：100 分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- **答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示**，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請衡酌作答。
- 答案卡請以 2B 鉛筆劃記，不可使用修正液（帶）塗改，未使用 2B 鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，後果由考生自負。
- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶書籍、紙張（應考證不得做計算紙書寫）、具有通訊、記憶、傳輸或收發等功能之相關電子產品或其他有礙試場安寧、考試公平之各類器材入場。
- **試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。**
- **試題採雙面列印，考生應注意試題頁數確實作答。**
- **違規者依本校招生考試試場規則及違規處理辦法處理。**

國立中山大學114學年度碩士班考試入學招生考試試題

科目名稱：離散數學與演算法【資工系資安碩班碩士班】　　題號：485001

※本科目依簡章規定「不可以」使用計算機(問答申論題)　　

*There are 10 problems in this test. Note that you should write down **detailed steps** for the solution to each problem; otherwise, no credits for that problem will be given.*

1. [10%] There are how many different ways we can write the number of 13 as a sum of positive integers, where the order of the summands is considered relevant.

2. [10%] If $n \in Z^+$ and n is composite, then there is a prime p such that $p|n$. Please prove it using the well-ordering principle.

3. For distinct primes, p, q, let $A = \{p^m q^n | 0 \le m \le 29, 0 \le n \le 41\}$.
   (a) [5%] What is |A|?
   (b) [5%] If $f: A \times A \to A$ is the closed binary operation defined by $f(a, b) = \gcd(a, b)$, does f have an identity element?

4. [10%] For $\Sigma = \{0,1\}$, let $A \subseteq \Sigma^*$, where $A = \{01, 1\}$. How many strings in A^* have length 6?

5. [10%] Find the generating function for the number of integer solutions to the equation $c_1 + c_2 + c_3 + c_4 = 23$, where $-2 \le c_1, -4 \le c_2, -3 \le c_3 \le 6, 0 \le c_4$.

6. [10%] If a fair die is rolled for 15 times, what is the probability that the sum of the rolls is 43?

7. [10%] Solve the following recurrence relation. $a_{n+2} - 5a_{n+1} + 6a_n = 5^n, n \ge 0, a_0 = 0, a_1 = 1$.

8. (Algorithm Points) [10%] Please describe the algorithm of Heap Sort and analyze its time complexity in detail.

9. (Algorithm Points) [10%] Given a knapsack with maximum capacity Q and a set L consisting of n items, where each item i has weight w_i and benefit value a_i. Please describe an algorithm in detail based on dynamic programming for packing the items in a knapsack to achieve a maximum total benefit value and analyze the time complexity of the algorithm. (w_i, a_i, n, and Q are integer values.)

10. (Algorithm Points) [10%] Please describe the algorithm of minimum spanning tree and analyze its time complexity in detail.