114ME02

國立臺北科技大學 114 學年度碩士班招生考試

系所組別：1120 機械工程系機電整合碩士班乙組

第一節　工程數學　試題

注意事項：

1.本試題共 5 題，每題 20 分，共 100 分。
2.不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3.全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. Solve for the following differential equation (20%).

$$y'' + 2y' + 2y = 3.5\sin 3x - 3\cos 3x, \text{and } y(0) = 0,\ y'(0) = 0.5.$$

2. Solve for the following ordinary differential equation (20%).

$$y' + \frac{1}{3}y = \frac{1}{3}(1 - 2x)y^4.$$

3. Find the inverse matrix of A, where $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1/2 & 1 \\ 1/3 & 0 & 0 \end{bmatrix}$ (20%).

4. Use the Laplace transform to solve the following equation. (20%)

$$y" + 2y' + 2y = \delta(t - 3), \text{ and } y(0) = y'(0) = 0.$$

5. Consider a suspension bridge subjected to uniform wind loading conditions. To analyze the dynamic behavior of this structure, we focus on the vertical displacement of the bridge deck, which can be mathematically modeled using the wave equation. In this analysis, we denote the vertical displacement as $y(x,t)$, where x represents the position along the bridge's length and t represents time. The bridge has a total length of L meters, and the structural properties of the deck determine the wave propagation speed c, which represents how quickly vibrations travel through the structure. The bridge is designed with fixed supports at both ends, constraining the vertical movement at these points. When the uniform wind load acts on the bridge deck, it creates an initial vertical velocity of magnitude A throughout the entire length of the structure. This initial condition, combined with the fixed boundary conditions, allows us to study how the bridge will respond to this sudden wind loading. The behavior of the bridge deck under these conditions can be described by a second-order partial differential equation, which relates the acceleration of any point on the deck to the spatial variation of displacement along its length.

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2},\ for\ 0 < x < L,\ t > 0$$

$$y(0,t) = y(L,t) = 0,\ for\ t \geq 0$$

$$y(x,0) = 0,\ \text{for } 0 \leq x \leq L$$

$$\frac{\partial y}{\partial t}(x,0) = g(x),\ \text{for } 0 \leq x \leq L$$

$$g(x) = A$$

Analyze the vertical vibration behavior of the bridge deck under these conditions. Consider how the displacement $y(x,t)$ evolves over time and how it varies along the length of the bridge. (20%)