

| 考試科目   | 微積分                                      | 系所別 | 企業管理研究所<br>(MBA 學位學程)乙組 | 考試時間 | 2 月 12 日(三) 第四節 |
|--|--|-----|-------------------------|------|-----------------|
| <p><b>Show all your work to earn the credits.</b></p> <p>1. (10 points) Evaluate: <math>\lim_{x \rightarrow \infty} x^{(\ln 2)/(1+\ln x)}</math></p> <p>2. (10 points) If <math>a, b</math> and <math>c</math> are constants such that</p> $\lim_{x \rightarrow 0} \frac{ax^2 + b(e^x - 1) + \sin(cx)}{2x^2 + 3x^5} = 1.$ <p>Find the value of <math>4a + b - c</math>.</p> <p>3. (10 points) Find the derivative <math>f'(x)</math> for the function</p> $f(x) = \frac{\sec x}{1 + \tan x} + \sum_{n=1}^{\infty} \int_{-nx^2}^{nx^2} (\sin u)^{2025} du.$ <p>4. (10 points) Find the derivative <math>g'(x)</math> for the function</p> $g(x) = \sqrt{x + \sqrt{x + \sqrt{x}}}.$ <p>5. (10 points) Evaluate: <math>\int_0^{\frac{\pi}{2}} \frac{\cos x \sin x}{3 + \cos^2 x} dx</math></p> <p>6. (10 points) Evaluate: <math>\int_0^1 x^2 \tan^{-1}(x) dx</math></p> <p>7. (10 points) Let <math>f(x) = xe^{-x^2}</math>. Find the 2025th derivative <math>f^{(2025)}(0)</math> of <math>f</math> at 0.</p> <p>8. A bus company offers sightseeing tours of downtown Taipei. One tour, priced at \$700 per person, had an average demand of about 1,000 customers per week. When the price was lowered to \$600, the weekly demand increased to about 1,200 customers. Let <math>x</math> represent the number of customers per week, and let <math>p</math> represent the price of a tour ticket. Assume that the demand equation is linear.</p> <p>(a) (3 points) Find the weekly demand equation.</p> <p>(b) (3 points) Find the weekly revenue function.</p> <p>(c) (4 points) Find the tour price that should be charged per person to maximize the total revenue each week.</p> |  |     |                         |      |                 |
| 備註   | <p>一、作答於試題上者，不予計分。</p> <p>二、試題請隨卷繳交。</p> |     |                         |      |                 |

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| <p>9. Students in a Calculus class get <math>x</math> hours of sleep in the two days leading up to the exam, where <math>x</math> lies in the range <math>0 \leq x \leq T</math>. The number of students who got between <math>a</math> and <math>b</math> hours of sleep is given by</p> $\int_a^b rxdx, \quad \text{for } 0 \leq a \leq b \leq T.$ <p>(a) (5 points) What fraction of the students got less than <math>T/2</math> hours of sleep?</p> <p>(b) (5 points) It is known that their scores, <math>G(x)</math>, are proportional to the amount of sleep <math>x</math> (in hours) they got, that is,</p> $G(x) = 100 \times \frac{x}{T}.$ <p>Find the (correctly weighted) average score in the class.</p> <p>10. Approximate the integral</p> $\int_0^4 \frac{1}{1+x^3} dx$ <p>(a) (5 points) using the Trapezoidal rule with four intervals.</p> <p>(b) (5 points) using the Simpson's rule with four intervals.</p> |                                       |       |                         |         |                   |
| 備 註  | <p>一、作答於試題上者，不予計分。<br/>二、試題請隨卷繳交。</p> |       |                         |         |                   |