

國立臺灣科技大學

114學年度碩士班招生

試題

系所組別：0330機械工程系碩士班丙組

科 目：工程數學

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(總分為 100 分；所有試題務必於答案卷內頁依序作答)

1. (10%) Saying that y is a function of x , solve the following initial value problem,

$$\frac{dy}{dx} = -4x^2y$$

$$y(0) = 2$$

2. (20%) Given an ODE,

$$\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + y = 30e^{-x}$$

- (a) Find the homogeneous solution (10%).
(b) Find the particular solution (10%).

3. (20%) Laplace transform is a useful tool for solving transient heat transfer problems. Let's define the Laplace transform $\mathcal{L}[f(t)] = \int_0^\infty f(t)e^{-st}dt$, where t is time, s is the Laplace transform variable.

- (a) Show that $\mathcal{L}(f') = s\mathcal{L}(f) - f(0)$ (5%).
(b) Show that $\mathcal{L}(f'') = s^2\mathcal{L}(f) - sf(0) - f'(0)$ (5%).
(c) Find the Laplace transform of $f(t) = t \sin at$, where a is a constant.
Hint: $\mathcal{L}(\cos at) = s/(s^2 + a^2)$ (10%).

4. (10%) Consider the plane curve $y = y(x)$, $0 \leq x \leq 1$, calculate the curvature of the curve $y = y(x)$ (5%). What would you get if $y' < 1$ (5%)?

5. (20%) Fourier transform is an important technique in engineering. Please answer the following questions regarding the Fourier transform:

- (a) Write down the definition of the Fourier transform $F(\omega)$ of the time-domain function $f(t)$ (5%).
(b) Write down the corresponding inverse Fourier transform theorem (No proof is needed!) (5%).
(c) Let

$$f(t) = \begin{cases} 1 & -T \leq t \leq T \\ 0 & |t| > T \end{cases}$$

calculate the corresponding Fourier transform $F(\omega)$ (5%).

- (d) Evaluate the following improper integral (5%):

$$\int_{-\infty}^{\infty} \frac{\cos(2\omega) \sin(3\omega)}{\omega} d\omega$$

6. (20%) Use the method of separation of variables to solve the following equilibrium equation:

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0 \quad 0 < r < 1, 0 < \theta < \frac{\pi}{2}$$

$$u_\theta(r, 0) = 0 \quad 0 \leq r \leq 1$$

$$u_\theta\left(r, \frac{\pi}{2}\right) = 0$$

$$u_r(1, \theta) = f(\theta) \quad 0 \leq \theta \leq \frac{\pi}{2},$$

where $f(\theta) = \cos(2\theta)$ (15%). What kind of constraint must be imposed on the function $f(\theta)$ if the problem is solvable? In other words, what would happen if $f(\theta) = \sin(2\theta)$? (5%)

