國立臺灣科技大學 114學年度碩士班招生

試題

系所組別:0310機械工程系碩士班甲組

科 目:工程數學

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1 \((15\%) \) Solve
$$x^2y'' - 3xy' + 3y = 2x^4e^x$$

$$2 \cdot (15\%)$$
 Solve $(x + 2y)dx + 2xdy = 0$, $y(1)=0$

3 \ (20%) Find the Laplace (or Inverse Laplace) transform of the following equation:

(1) (10%)
$$\mathcal{L}^{-1}\left\{\frac{6s+3}{s^2+4}\right\}$$

(2) (10%)
$$\mathcal{L}\{e^{2t} * \sin(t)\}$$

4 \ (20%)

- (1) (10%) Compute the directional derivative of the field $f(x,y,z) = x^3 + y^2z$ in the direction of the vector $2\bar{i} + \bar{k}$ at the point P: (1, 1, 1).
- (2) (10%) Evaluate the following integral from A:(0, 1, 2) to B:(1, 1, 7).

$$I = \int_{C} \left(3x^2 dx + 2yz dy + y^2 dz \right)$$

5 \ (10%) Evaluate

$$\int_{-\infty}^{\infty} \frac{x^2}{\left(x^2 + a^2\right)\left(x^2 + b^2\right)} dx$$

where a and b are positive constants.

6 \ (20%) Solve the following partial differential equation

$$\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial x^2} + 2$$

Boundary conditions : $\theta(l,t) = 0$, $\frac{\partial \theta(0,t)}{\partial x} = 0$,

Initial conditions : $\theta(x,0) = 0$

Starting from separation of variables according to the following steps

- (1) (10%) assume separation of variables $\theta(x,t) = \varphi(x,t) + \phi(x)$ and solve $\phi(x)$.
- (2) (10%) solve $\varphi(x)$ using a further separation of variable and determine the full answer of $\theta(x,t)$.

