國立中山大學114學年度
碩士班考試入學招生考試試題

科目名稱：工程數學【光電系碩士班】

—作答注意事項—

考試時間：100分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- **答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示**，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請衡酌作答。
- 答案卡請以2B鉛筆劃記，不可使用修正液（帶）塗改，未使用2B鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，後果由考生自負。
- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶書籍、紙張（應考證不得做計算紙書寫）、具有通訊、記憶、傳輸或收發等功能之相關電子產品或其他有礙試場安寧、考試公平之各類器材入場。
- **試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。**
- **試題採雙面列印，考生應注意試題頁數確實作答。**
- **違規者依本校招生考試試場規則及違規處理辦法處理。**

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※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

1. Let A be a 3×3 matrix with det(A)=3. How many solutions does the homogeneous system Ax=0 have? Why? (5%)

2. Let **A** have eigenvalues $\lambda_1, \ldots, \lambda_n$ and suppose that **P** diagonalizes **A**. Show that for any positive integer k, **P** diagonalizes $\mathbf{A}^k$ and determine $\mathbf{P}^{-1}\mathbf{A}^k\mathbf{P}$. (10%)

3. (a) Please determine the rank of the matrix A as a function of k. (10%)

$$A = \begin{bmatrix} 5-k & 4 & 2 \\ 4 & 5-k & -2 \\ -2 & -2 & 3-2k \end{bmatrix}$$

(b) If k = 2, Find inverse matrix A^{-1}. (5%)

4. Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$
Carry out the diagonalization process, if possible. (5%)

5. Two student solve the same initial value problem $y''(x)+ay'(x)+by(x)=0$ with given initial conditions $y(0)=A$ and $y'(0)=B$. Using wrong constants for b and B, one student got the solution $y_A(x)=e^{-2x}(cos3x+2sin3x)$. Using wrong constants for a and A, the other one got the solution $y_B(x)=-3e^{x}+2e^{3x}$. Find the correct constants for a, b, A and B and solve the initial value problem. (10%)

6. Solve the given differential equation (10%)

$$6x^2y + 12xy + y^2 + (6x^2 + 2y)y' = 0$$

7. It is known that $y_h = e^{-2t} + e^{-t}, y_p = \frac{1}{2}e^{-3t}$ is the solution of ordinary differential equation $y'' + Ay' + By = C$ What is the coefficient A*B*C ? (10%)

8. Verify the divergence theorem $\vec{F} = (xy - 1)\vec{\imath} + yz\,\vec{\jmath} + xz\,\vec{k}$, and D is the region bounded by the unit cube defined $0\le x\le 1, 0\le y\le 1, 0\le z\le 1$. (10%)

9. If $z(x,y) = 3000 - 2x^2 - 9y^2$ [meter] gives the elevation of a mountain at sea level, what is the direction of steepest ascent at $P(4,1)$? (10%)

10. A small robotic toy is programmed to follow a specific path on the floor of a circular room. The vector field affecting the toy's movement is given by $\vec{F} = -3xy\,\vec{\imath} + 2y\vec{k}$.
Calculate the line integral of $\vec{F}$ along the path C.
(C is the semicircle $x^2 + z^2 = 4,\ y = 1,\ z \ge 0$, oriented from (2,1,0) to (−2,1,0). (15%)