第一大題「單選題」，請依題號作答於「答案卡」（請勿作答於試卷之選擇題作答區），未作答於答案卡者，該大題不予計分；第二大題爲「非選擇題」，請作答於「試卷」之非選擇題作答區。

一、單選題

1. (5%) The convolution of $x(t)$ and $y(t)$ is denoted by $x(t) * y(t)$. Find the continuous-time Fourier transform of $e^{-|t|} * \cos(20\pi t)$.

   (A) $\frac{1}{1+(\omega-20\pi)^2}+\frac{1}{1+(\omega+20\pi)^2}$.
   (B) $\frac{2}{1+(\omega-20\pi)^2}+\frac{2}{1+(\omega+20\pi)^2}$.
   (C) $\frac{2\pi}{1+400\pi^2}\left(\delta(\omega-20\pi)+\delta(\omega+20\pi)\right)$.
   (D) $\frac{-j2\pi}{1+400\pi^2}\left(\delta(\omega-20\pi)-\delta(\omega+20\pi)\right)$.
   (E) $\frac{2\pi}{1+400\pi^2}\left(\delta(\omega-10\pi)+\delta(\omega+10\pi)\right)$.

2. (5%) We consider the following statements:

   {I} The continuous-time system $\mathcal{S}$ is linear time-invariant if and only if $e^{j\omega_0 t}$ is an eigenfunction of $\mathcal{S}$ for all $\omega_0 \in \mathbb{R}$.
   {II} The discrete-time signal e^{jn} is periodic.
   {III} If a system $\mathcal{S}$ is memoryless, then $\mathcal{S}$ is causal.

   Select a choice that best describes these above statements.

   (A) {I} is true, {II} is false, and {III} is false.
   (B) {I} is true, {II} is true, and {III} is true.
   (C) {I} is true, {II} is false, and {III} is true.
   (D) {I} is false, {II} is true, and {III} is true.
   (E) {I} is false, {II} is false, but {III} is true.

3. (6%) Let $\{a_k\}_{k\in\mathbb{Z}}$ be the Fourier series coefficients of a periodic signal $x(t)$ with period T. We consider the truncated signal $x_N(t)$ by $x_N(t) \triangleq \sum_{k=-N}^{N} a_k e^{j\frac{2\pi k t}{T}}$, where N is a positive integer. The energy in the error is defined as $E_N \triangleq \int_0^T |x(t)-x_N(t)|^2 \, dt$. We have the following statements:

   {I} $\lim_{N\to\infty} x_N(t) = x(t)$ for all $t \in \mathbb{R}$.
   {II} $\lim_{N\to\infty} E_N = 0$.
   {III} The Fourier series coefficients of $x(-t)$ are a_{-k}.

   Select a choice that best describes these above statements.

   (A) {I} is true, {II} is true, and {III} is false.
   (B) {I} is true, {II} is true, and {III} is true.
   (C) {I} is false, {II} is true, and {III} is true.
   (D) {I} is true, {II} is false, and {III} is true.
   (E) {I} is false, {II} is false, and {III} is true.

4. (5%) We consider a continuous-time, stable, linear time-invariant system with system function

$$H(s) = \frac{1}{(s^2+6s+18)(s+2)(s+1)(s-4)}.$$

   What is the region of convergence of $H(s)$?

   (A) $\{s \mid \mathrm{Re}\{s\} < -3\}$.
   (B) $\{s \mid -3 < \mathrm{Re}\{s\} < -2\}$.
   (C) $\{s \mid -2 < \mathrm{Re}\{s\} < -1\}$.
   (D) $\{s \mid -1 < \mathrm{Re}\{s\} < 4\}$.
   (E) $\{s \mid \mathrm{Re}\{s\} > 4\}$.

5. (6%) We consider the discrete-time signal $x[n] \triangleq \frac{e^{jn^2}}{\sqrt{(|n|+1)(|n|+2)}}$, where $n \in \mathbb{Z}$. The discrete-time Fourier transform of $x[n]$ is $X(e^{j\omega})$. Find the value of the integral $\int_0^{4\pi} |X(e^{j\omega})|^2 \, d\omega$.

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(A) π. (B) 2π. (C) 3π. (D) 4π. (E) 6π.

6. (6%) The discrete-time unit step sequence is denoted by $u[n]$. We consider the discrete-time signal

$$x[n] = a_1^n u[n] + a_2^n u[n] + a_3^n u[-n] + a_4^n u[-n] + a_5^{-|n|} + a_6^{-|n|},$$

where the parameters

$$a_1 = (1+j)/2, \quad a_2 = (2-5j)/8, \quad a_3 = -2+j, \quad a_4 = -4-j, \quad a_5 = 3/2, \quad a_6 = 4,$$

and $n \in \mathbb{Z}$. The z-transform of $x[n]$ is $X(z)$, where $z \in \mathbb{C}$. For $\ell = 1, 2, 3$, we define another discrete-time signal $y_\ell[n]$ by $y_\ell[n] \triangleq \int_{C_\ell} X(z) z^{n-1} \mathrm{d}z$, where the contour C_ℓ evaluated counterclockwise are given as

$$C_1 \triangleq \{z \mid |z| = 0.6\}, \qquad C_2 \triangleq \{z \mid |z| = 0.8\}, \qquad C_3 \triangleq \{z \mid |z| = 1.2\},$$

The error is defined as $\mathcal{E}_\ell \triangleq \sum_{n=-\infty}^{\infty} |j2\pi x[n] - y_\ell[n]|^2$. Which of the following relation is true?

(A) $\mathcal{E}_1 = \mathcal{E}_2 = \mathcal{E}_3$. (B) $\mathcal{E}_1 > \mathcal{E}_2 = \mathcal{E}_3$. (C) $\mathcal{E}_1 = \mathcal{E}_2 > \mathcal{E}_3$. (D) $\mathcal{E}_2 > \mathcal{E}_1 = \mathcal{E}_3$. (E) $\mathcal{E}_2 = \mathcal{E}_3 > \mathcal{E}_1$.

7. (6%) A causal linear time-invariant discrete-time system $\mathcal{S}$ has the transfer function

$$H(z) \triangleq \frac{(z+2)(z+4)}{(z-1)(z-2)(z-3)}.$$

The impulse response of $\mathcal{S}$ is $h[n]$. The input signal is $x[n]$ while the output signal is $y[n]$. We consider the following statements:

{I} The system $\mathcal{S}$ is unstable.

{II} $\lim_{n\to\infty} \frac{h[n]}{3^n} = 0$.

{III} The system $\mathcal{S}$ can be described by the difference equation

$$y[n] - 6y[n-1] + 11y[n-2] - 6y[n-3] = x[n] + 9x[n-1] + 8x[n-2].$$

Select a choice that best describes these above statements.

(A) {I} is true, {II} is true, and {III} is true.
(B) {I} is true, {II} is false, and {III} is false.
(C) {I} is true, {II} is false, and {III} is true.
(D) {I} is true, {II} is true, and {III} is false.
(E) {I} is false, {II} is true, and {III} is false.

8. (5%) Let $x(t)$ be a bandlimited signal. Which of the following signals has the same bandwidth as $x(t)$?

(A) $8x(t-3)$. (B) $x(t^2-5)$. (C) $\sin(x(t))$. (D) $|x(t)|^2$. (E) $x(2t+4)$.

9. (6%) Let the $x_c(t)$ be a continuous-time signal whose continuous-time Fourier transform is $X_c(j\omega)$. It is assumed that $X_c(j\omega) = 0$ if $|\omega| > 1000\pi$. The discrete-time signal $x[n] \triangleq x_c(nT_1)$ for some $T_1 > 0$. We define the reconstructed signal as

$$x_r(t) \triangleq \frac{T_1}{\pi} \sum_{n=-\infty}^{\infty} x[n] \frac{\sin\left(\frac{2\pi}{T_2}(t - nT_1)\right)}{t - nT_1},$$

where $T_2 > 0$. Select a sufficient condition such that $x_r(t) = x_c(t)$ for all $x_c(t)$.

(A) $T_1 = \frac{1}{800}, T_2 = \frac{1}{400}$.
(B) $T_1 = \frac{1}{800}, T_2 = \frac{1}{500}$.
(C) $T_1 = \frac{1}{1200}, T_2 = \frac{1}{400}$.
(D) $T_1 = \frac{1}{1200}, T_2 = \frac{1}{600}$.
(E) $T_1 = \frac{1}{1200}, T_2 = \frac{1}{800}$.

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Part 2 : Please simplify your answers as much as possible and write down detailed steps

10. Two equal-probability messages with indexes j are mapped into symbol waveforms as

$$S(t,j) = \begin{cases} d\sqrt{2}\text{sinc}(2t) & \text{if } j=1 \\ d\sqrt{2}\text{sinc}(2t)\sin(\pi t) & \text{if } j=2 \end{cases}$$

where d is a positive real number. Assume there is an additive noise in the channel.
(a) (5%) Assume the noise is an additive white Gaussian noise with power spectrum $S_N(f) = \sigma^2$, please find the decision regions of the optimal receiver, which minimizes the symbol error probability.
(b) (8%) Please find the symbol error probability of your answer in (a)
(c) (12%) Repeat (a) for the case where the additive noise is still Gaussian but with autocorrelation function $\frac{\sigma^2}{2}\exp(-|\tau|)$.

11. In binary $(n;k)$ linear code, a k-bit message is represented as a $1\times k$ binary vector $\mathbf{b}$, and the n-bit binary codeword $\mathbf{c}_b$ is encoded from message $\mathbf{b}$ via a linear transformation as

$$\mathbf{c}_b = \mathbf{b}\mathbf{G}$$

where $k\times n$ binary matrix $\mathbf{G}$ the "generator matrix" of this code and the matrix multiplication is in binary finite field.
(a) (5%) We collect all possible codewords generated from $\mathbf{G}$ as a codebook $\mathcal{C}$, show that for any two codewords $\mathbf{c}_{b,i}\in\mathcal{C}, i=1,2$, the finite-field (modulo-2) addition of them is a valid binary codeword belongs to $\mathcal{C}$
(b) (10%) Prove that for any binary $(n;k)$ linear code that can correct up to E binary errors in its codeword, integers n,k,E must satisfy the inequality

$$2^{n-k} \geq \sum_{t=0}^{E}\binom{n}{t}$$

where combinatorial number $\binom{n}{t}$ is defined as $n!/(t!(n-t)!)$.
(c) (10%) Let $E=0.1n$, from (b) prove that when $n\to\infty$, the maximum possible rate $\log_2(k/n)$ approaches $1-H(0.1)$ where $H(x) = -(x\log_2(x)+(1-x)\log_2(1-x))$. You can use the following large n approximation : $n! \approx \sqrt{2\pi n}(n/e)^n$ where e is the Euler's number.

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