考試科目	統計學	系所別	風險管理與保險學系 精算科學組	考試時間	2 月 12 日 (三) 第四節

1. (30 pts) Please explain the following items.

   (a) (10 pts) Likelihood Ratio Tests

   (b) (10 pts) Exponential memoryless

   (c) (10 pts) Convergence in distribution

2. (10 pts) An insurance company models the number of claims it receives in a year using the following distribution. Let X represent the number of claims filed for natural disasters (e.g., earthquakes, hurricanes) in a given year, where:

$$f_X(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad x = 0, 1, 2, \ldots,$$

   with $\lambda = 2$. For each claim received, there is a 50% chance that the claim is for damages caused by flooding. Let Y represent the number of flood-related claims in a year. What is the distribution of Y?

   (a) $f_Y(y) = \frac{e^{-1}2^y}{y!}$

   (b) $f_Y(y) = \frac{e^{-2}}{y!}$

   (c) $f_Y(y) = \frac{e^{-0.5}0.5^y}{y!}$

   (d) $f_Y(y) = \frac{e^{-1}0.5^y}{2y!}$

   (e) None of the above.

3. (10pts) Which of the following statements about the Central Limit Theorem (CLT) is incorrect?

   (a) The CLT describes the sampling distribution of population mean.

   (b) When using the CLT, the population from which the data is drawn does not need to follow a normal distribution.

   (c) When using the CLT, the population from which the data is drawn does not need to consist of continuous data.

   (d) When using the CLT, the sample size must be sufficiently large.

   (e) None of the above.

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4. (10pts) Let X have a Poisson distribution with mean λ. Find a transformation $u(X)$ so that $\text{Var}[u(X)]$ is free of λ, for large values of λ.

   Hint: Use Tylor Expansion $u(X) \approx u(\lambda) + [u'(\lambda)](X-\lambda) + \frac{u''(\lambda)}{2}(X-\lambda)^2$, and higher terms can be neglected when λ is large.

5. (20pts) Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables with cumulative distribution function $F(\cdot)$, and consider the statistic

$$F_n(x) = \frac{1}{n}\sum_{i=1}^{n} I(X_i \leq x),$$

   where $I(\cdot)$ is the indicator function.

   (a) (5 points) Prove that $\mathbb{E}[F_n(x)] = F(x)$.

   (b) (10 points) Derive the variance of $F_n(x)$.

   (b) (5 points) Determine whether $F_n(x)$ is the UMVUE (Uniformly Minimum Variance Unbiased Estimator) of $F(x)$. Justify your answer.

6. (20 pts) If $X_1, \ldots, X_n$ is a random sample from a geometric distribution

$$P(X = x) = \theta(1-\theta)^{x-1} I_{\{1,2,\ldots\}}(x),$$

   and assume θ follows a uniform prior on $(0, 1)$.

   (a) (10 pts) Find the posterior distribution of θ.

   (b) (10 pts) Determine the Bayes estimator of θ under squared error loss, where the loss function is defined as

$$L(\theta, \delta(X_1, \ldots, X_n)) = (\delta(X_1, \ldots, X_n) - \theta)^2.$$

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。