

考試科目	微積分	系所別	風險管理與 保險學系	考試時間	2 月 12 日 (三) 第 2 節
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Problem 1 (20 points) (10% each)

Evaluate the integrals.

(a)

$$\int_0^3 \frac{x^2}{\sqrt{x+1}} dx.$$

(b)

$$\int \frac{3x^3 - 2x - 2}{x^2(x^2 + 1)} dx.$$

Problem 2 (20 points) (10% each)

Find the following limits if they exist.

(a)

$$\lim_{x \rightarrow 0^+} (1 - \cos x)^{\frac{1}{\ln x}}.$$

(b)

$$\lim_{x \rightarrow 1} \left(\frac{1}{\ln x} - \frac{1}{x-1} \right).$$

Problem 3 (15 points)(15%)

Let

$$F(x) = \int_0^{x^2} \cos(t^2 + t) dt.$$

(a) (6%) Find $F'(x)$.

(b) (9%) Find

$$\lim_{x \rightarrow 0} \frac{F(x)}{x^2}.$$

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註

- 一、作答於試題上者，不予計分。
二、試題請隨卷繳交。

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Problem 4 (15 points)(5% each)

- (a) Derive the MacLaurin series of

$\tan^{-1}x.$

- (b) Find the value of
- $a \in \mathbb{R}$
- such that the limit

$$\lim_{x \rightarrow 0} \frac{\sin(ax) - \sin x - \tan^{-1}x}{x^3}$$

is finite.

- (c) Evaluate the above limit.

Problem 5 (15 points)(15%)

Find the maximum and the minimum of the function

$f(x, y) = 3x^2 - 2y^2$

on the curve

$2x^2 - 2xy + y^2 = 1.$

Problem 6 (15 points) (5% each)

Let $f(x) = xe^x$.

- (a) Find the Taylor series for
- $f(x)$
- at
- $x = 0$
- .

- (b) Calculate

$$\int_0^x te^t dt$$

and find its Taylor series at $x = 0$.

- (c) Find the sum

$$\sum_{n=0}^{\infty} \frac{1}{n!(n+2)}.$$

備

註

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