國立政治大學 114 學年度 碩士班暨碩士在職專班 招生考試試題

考 試 科 目	數理統計學	系 所 別	統計學系	考試時間	2 月 12 日（三）第 二 節

1. (15%) An electronic device has lifetime denoted by T. The device has value $V = 5$ if it fails before time $t = 3$; otherwise, it has value $V = 2T$. If T has probability density function (PDF) $f_T(t) = \frac{2}{3}\exp\left(-\frac{2t}{3}\right), t > 0$, find the cumulative distribution function (CDF) of V.

2. Let $X_1, X_2, \ldots, X_n$ be independently and identically distributed (i.i.d.) random samples from a gamma distribution with shape parameter $\alpha > 0$ and scale parameter $\beta > 0$, having the PDF

$$f_G(x) = \frac{x^{\alpha-1}\exp(-x/\beta)}{\Gamma(\alpha)\beta^{\alpha}}, x > 0, \alpha > 0, \beta > 0.$$

   i. (15%) Find the maximum likelihood estimator of ψ, denoted by $\hat{\psi}$, where $\psi = \alpha\beta$.

   ii. (15%) Solve for the smallest sample size such that the variance of $\hat{\psi}$ is smaller than a constant $c > 0$. The answer should be expressed in terms of α, β and c.

3. Let $X_1, X_2, \ldots, X_n$ be i.i.d. random samples from a Poisson distribution with mean $\lambda > 0$, denoted as $Poisson(\lambda)$, having the probability mass function

$$P_X(i) = \frac{\lambda^i \exp(-\lambda)}{i!}, i = 0,1,2, \ldots.$$

   i. (15%) Show that $Y = \sum_{i=1}^{n} X_i \sim Poisson(n\lambda)$ and Y is sufficient for λ.

   ii. (10%) Using the fact that "$\Pr(Y \le y_0) = \Pr(Z > 2n\lambda)$, where $Z \sim \chi^2_{2y_0}$ follows a chi-square distribution with $2y_0$ degrees of freedom," show that the $100(1-\alpha)\%$ confidence interval for λ is $\left(\frac{1}{2n}\chi^2_{2y_0;1-\frac{\alpha}{2}}, \frac{1}{2n}\chi^2_{2y_0+2;\frac{\alpha}{2}}\right)$ when $Y = y_0 > 0$ is observed, where $\chi^2_{p;\alpha}$ is the chi-square α^{th} quantile for upper tail probability on p degrees of freedom.

4. Let $W_1, W_2, \ldots, W_n$ be i.i.d. random samples from a truncated normal distribution $TN(\mu, \sigma, a)$ with the PDF

$$f_W(w) = \frac{\frac{1}{\sigma}\phi\left(\frac{w-\mu}{\sigma}\right)}{1-\Phi\left(\frac{a-\mu}{\sigma}\right)}, w > a, \sigma > 0, \mu \in \boldsymbol{R}(real\ numbers), a \in \boldsymbol{R},$$

   where ϕ and Φ are the PDF and CDF of the standard normal distribution, respectively.

   i. (10%) Find a minimal sufficient statistic for the parameter a.

   ii. (10%) Give a $100(1-\alpha)\%$ rejection region for the null hypothesis $H_0: a > 0$.

   iii. (10%) If $W \sim TN(\mu, \sigma, a)$ and $U|W = w \sim Poisson(\lambda w)$ for any $w \ge a$, calculate the expectation value for W and U.

備 註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。