招 生 學 年 度	九十九	招 生 類 別	轉學招生考試
系 所 班 別	資訊工程學系三年級		
科　　　　　目	離散數學		
注 意 事 項	禁止使用掌上型計算機		

Discrete Mathematics

1. Let X be a set with n elements, $n > 3$. Determine the sizes of each of the following sets in terms of n, and give a big-Θ estimate for each answer.

    (a) (10%) $P(X)$ the power set of X.

    (b) (10%) The set of all one-to-one correspondences $X \to X$.

    (c) (10%) The set of all subsets of X of size 3.

2. (10%) Prove the following statement by contraposition.
   Let x be an integer. If $x^2 + x + 1$ is even, then x is odd.

3. (10%) Prove the following statement by contradiction.
   Let x and y be integers. If $3x + 5y = 153$, then at least one of x and y is odd.

4. Let $S = \{0, 1, 2, 3, 4, 5\}$, and let $P(S)^*$ be the set of all nonempty subsets of S. Define a function $f : P(S)^* \to S$ by $f(H) =$ the largest element in H for any nonempty subset H of S.

    (a) (10%) Is f one-to-one? Why or why not?

    (b) (10%) Does f map $P(S)^*$ onto S? Why or why not?

5. An integer n is even if $n = 2k$ for some integer k.
   Define a relation R on $\mathbf{Z}$ by $x \, R \, y$ if $x + y$ is even.

    (a) (10%) Show that R is an equivalence relation.

    (b) (10%) Describe the equivalence classes formed by this relation.

6. (10%) Suppose that 100 lottery tickets are given out in sequence to the first 100 guests to arrive at a party. Of these 100 tickets, only 12 are winning tickets. Find the maximum k such that there must be a streak of at least k losing tickets in a row.