國立臺灣師範大學 112 學年度碩士班招生考試試題

科目：數值分析　　　　　　　　　　適用系所：數學系

注意：1.本試題共 1 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則依規定扣分。

1. Consider the equation $x + 3x^3 + x^7 = 1$.

   (a) (10 points) Show that this equation has a real solution.

   (b) (20 points) Explain how you solve this equation numerically. You can answer in Chinese.

2. Let $x_n = \frac{n}{2^n}$.

   (a) (10 points) Find the limit of x_n.

   (b) (15 points) Find the convergence order of the sequence $\{x_n\}$.

3. Let $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 \\ 1 & 2 & 3 & 4 \end{bmatrix}$.

   (a) (15 points) Find the LU decomposition of A. This means you need to find a lower triangular matrix L with diagonal entry 1 and an upper triangular matrix U, such that $A = LU$.

   (b) (10 points) Show that A is a positive definite matrix. This means $\mathbf{x}^\top A \mathbf{x} > 0$ for all $\mathbf{x} \in \mathbb{R}^4 \backslash \{\mathbf{0}\}$.

4. (20 points) Consider the following algorithm.

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1:  Input a 4x4 upper triangular matrix A and an ℝ⁴ vector b;
2:
3:  n=4;
4:  x(n)=b(n)/A(n,n);
5:
6:  for i=n-1 to 1
7:    x(i)=(b(i)-(A(i,i+1)*x(i+1)+...+A(i,n)*x(n)))/A(i,i);
8:  end
9:  Output x
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What is the output of this algorithm? Explain your reason.