## 台灣聯合大學系統112學年度碩士班招生考試試題

## 類組:電機類 科目:離散數學(300C)

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計算題(計算題應詳列計算過程,無計算過程者不予計分)

- 1. [20 points] Dr. DM has created a new nation called NYCU. His job is to find out what denominations to issue new currency, e.g., 1 dollar, 5 dollars, 10 dollars in Taiwan. Dr. DM wants to reduce costs by only printing a minimal number of types of currency, and the currency of value less than 5 dollars will not be allowed.
- (a) [10 points] What is the minimum number n of integer values  $v_1, v_2, \dots, v_i, \dots, v_n$  (where  $v_i \ge 5$  for all i) such that all positive integers greater or equal to 5 can be expressed as linear combinations of them? What is the precise condition that such a minimal set  $v_1, v_2, \dots, v_i, \dots, v_n$  has to satisfy? Prove.
- (b) [10 points] Suppose Dr. DM sets the lowest possible prize of items at c dollars, for some  $c \ge 50$ . What is the minimum number of integer values  $v_1, v_2, \dots, v_i, \dots, v_n$  (where  $v_i \ge 5$  for all i) that Dr. DM can get away with so as to cover all cost greater or equal to c now? Prove.
- 2. [16 points] After issuing the currency in NYCU, Dr. DM holds and attends a lot of cocktail parties. He observed that there are two that know the same number of party attendees when the party is with at least two people.

  (a) [6 points] Prove or disprove the observation.
- (b) [10 points] Prove that at a cocktail party with ten or more people, there are either three mutual acquaintances or four mutual strangers.
- **3.** [15 points] To better understand the nation, Dr. DM builds the social relationship in NYCU nation as a simple graph G and finds out that G is with n vertices and k connected components.
- (a) [6 points] What is the minimum possible number of edges of G? Prove.
- (b) [9 points] What is the maximum possible number of edges of G? Prove.
- **4.** [12 points] Dr. DM realizes that Mathematics is the key to all sciences. He starts to study Mathematics and finds some interesting numbers called perfect numbers. A perfect number n equals the sum of its divisors (including 1, but not n itself). For example, 6 is a perfect number since 6 = 1 + 2 + 3. Prove that if  $(2^p-1)$  is prime, then  $(2^{p-1})(2^p-1)$  is a perfect number.
- 5. [5 points] After several years, Dr. DM is very sharp to Mathematics. When he sees the following number:

he immediately finds a divisor that is not a multiple of 3 and ranges in [2,99]. Can you also find this divisor without actually factorizing this number?

注意:背面有試題

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**6.** [10 points] Dr. DM represents a relation  $\bigstar$  on the set of functions from positive integer  $\mathbb{N}^+$  to  $\mathbb{R}$ , such that  $f \bigstar g$  if and only if f = O(g). Is  $\bigstar$  an equivalence relation? A partial order? A total order? Briefly explain your answer.

- 7. [10 points] Suppose a and b are relatively prime integers greater than 1. Please help Dr. DM prove that Euler's function is multiplicative. In other words,  $\phi(ab) = \phi(a)\phi(b)$ . Note that Euler's Totient function  $\phi(n)$  for an input n is the count of numbers in  $1, 2, 3, \dots, n-1$  that are relatively prime to n, *i.e.*, the numbers whose GCD (Greatest Common Divisor) with n is 1.
- **8.** [12 points (4 each)] After thousands of years, NYCU nation has developed many family trees. Let P denote the set of all people who ever lived. Parent(x, y) is true if and only if x is the parent of y. Ancestor(x, y) is true if and only if x has a light with y. Rewrite in the language of mathematical logic. The operations are limited to  $\forall$ ,  $\exists$ ,  $\neq$ ,  $\in$ ,  $\rightarrow$ ,  $\land$ ,  $\lor$ ,  $\leftrightarrow$ , and  $\neg$ .

(a) All people have two parents.

- (b) An ancestor of a person is one of the person's parents or the ancestor of (at least) one of the person's parents. Rewrite this definition using the language of mathematical logic. Specifically, you need to provide a necessary and sufficient condition for the predicate Ancestor(x, y) to be true. (Note that you can inductively use the  $Ancestor(\cdot, \cdot)$  predicate in the condition itself.)
- (c) Someone has a fight with his/her ancestor but does not have a fight with his/her offspring.

注意:背面有試題