112AT01

國立臺北科技大學 112 學年度碩士班招生考試

系所組別：1501、1502 自動化科技研究所

第一節　工程數學　試題

注意事項：

1. 本試題共 5 題，共 100 分。
2. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. (25%) Prove or disprove the following statements in detail (do not only answer True or False but a proof or counter-example is also needed):
   (1) (5%) Let T be a linear transformation. If $\{v_1, v_2, v_3\}$ is linearly dependent, then $\{T(v_1), T(v_2), T(v_3)\}$ is also linearly dependent.
   (2) (5%) $n\times n$ matrix A has n distinct eigenvalues if and only if A is diagonalizable.
   (3) (5%) Assume the matrix inverse exist, $\left(A^{-1}+B^{-1}\right)^{-1}=A\left(A+B\right)^{-1}B$
   (4) (5%) The $cos(x)$ and $sin(x)$ are linearly dependent.
   (5) (5%) Consider $Ax=b$ where A is $m\times n$. If the rank of matrix A is n, then there is a solution.

2. (20%) $A=\begin{bmatrix} 4 & 1+i \\ 1-i & 4 \end{bmatrix}$, $B=e^{A}$
   (1) Find the eigenvalues and eigenvectors of A, and $det(A)$. **(10%)**
   (2) Find the eigenvalues and eigenvectors of B, and $det(B)$. **(10%)**

3. (20%) Find the general solution of

$$y^{(4)}+11y^{(3)}+36y''+16y'-64y=-3e^{-4x}+2\cos(2x)$$

4. (20%) Solve the following initial value problem by Laplace transform:

$$\begin{cases} \dfrac{dx}{dt}=x-5y & x(0)=2 \\ \dfrac{dy}{dt}=-3x-7y & y(0)=2 \end{cases}$$

5. (15%) Let $S=Span\left[(1\ \ 3\ \ 1\ \ 1)^T, (1\ \ 1\ \ 1\ \ 1)^T, (-1\ \ 5\ \ 2\ \ 2)^T\right]$ be a subspace of R^4, and let $b=(4,-1,5,1)^T$
   (1) (10%) Find an orthonormal basis for S.
   (2) (5%) Use your answer in (1) to find the projection p of b onto S.