



1. (15%) Solve the general solution of the following differential equations:
 - (1) $y' = 5\sin 3x$ (5%)
 - (2) $y' + y = e^{5x}$ (5%)
 - (3) $\frac{dy}{dx} = (x + y + 3)^2$ (5%)
2. (15%) Use the Laplace transform or inverse transform to solve the given problems:
 - (1) $f(t) = te^{4t} + e^{2t}\sin t$; 求 $F(s) = \mathcal{L}[f(t)]$ (5%)
 - (2) $F(s) = \frac{e^{-2s}}{s(s-1)}$; 求 $f(t) = \mathcal{L}^{-1}[F(s)]$ (5%)
 - (3) $f(t) = \int_0^t \sin \tau \cos(t - \tau) d\tau$; 求 $F(s) = \mathcal{L}[f(t)]$ (5%)
3. (10%) Use Laplace transform to solve the given initial-value problem:
 $y'' - y' = e^t \cos t, y(0) = 0, y'(0) = 0$
4. (10%) Solve the given Cauchy-Euler differential equation:
 $2x^2y'' + 5xy' + y = x^2 - x$
5. (15%) Show that (a) the set V of all symmetric 2×2 matrices is a vector space, and find the dimension of V . (b) Find a basis of V consisting of invertible matrices.
6. (15%) Compute the rank of matrix $A = \begin{bmatrix} 1 & 2 & 2 & -1 \\ 3 & 6 & 5 & 0 \\ 1 & 2 & 1 & 2 \end{bmatrix}$ and find the bases for the row space and column space of A .
7. (10%) Using Gram-Schmidt orthogonalization algorithm, find an orthogonal basis of the row space of $A = \begin{bmatrix} 1 & 1 & -1 & -1 \\ 3 & 2 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$
8. (10%) Compute A^8 using block multiplication, where $A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 1 & -1 & -2 \end{bmatrix}$.
 Hint: $A = \begin{bmatrix} X & 0 \\ Y & Z \end{bmatrix}$, $X = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, $Y = [1 \quad -1]$, $Z = [-2]$