# 國立臺北大學112學年度碩士班一般入學考試試題

系（所）組別：金融與合作經營學系

科　　目：統計學



☑可 ☐不可使用計算機

1. (15%) Let $f(x;\theta) = (1/\theta)x^{(1-\theta)/\theta}$, $0 < x < 1$, $0 < \theta < \infty$.
   (1)(10%) Show that the maximum likelihood estimator of $\theta$ is $\hat{\theta} = -(1/n)\sum_{i=1}^{n} \ln(X_i)$.
   (2)(5%) Show the $E(\hat{\theta}) = \theta$ and thus $\hat{\theta}$ is an unbiased estimator of $\theta$.

2. (10%) Let $X$ have an exponential distribution with mean $\theta > 0$. Show that $P(X > x + y | X > x) = P(X > y)$.

3. (15%) Let $X_1$, $X_2$ be a random sample of size $n$=2 from a distribution with p.d.f. $f(x)=4x^3$, $0 < x < 1$, zero elsewhere. Given the ratio $Y = X_1/X_2$, find
   (1)(10%) E($Y$). (Round the answer to 4 decimal places)
   (2)(5%) Var($Y$). (Round the answer to 4 decimal places)

4. (10%) A die was cast $n$ = 120 independent times and the following data resulted:

| Spots up | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Frequency | b | 20 | 20 | 20 | 20 | 40-b |

   If we use a chi-square test, for what values of b would the hypothesis that the die is unbiased be rejected at the 0.025 significance level?
   ($\chi^2_{(5),\alpha=0.025} = 12.8$; $\chi^2_{(6),\alpha=0.025} = 14.4$; $\chi^2_{(5),\alpha=0.05} = 11.1$; $\chi^2_{(6),\alpha=0.05} = 12.6$)

5. (15%)An OLS regression has the following data: $\bar{X} = 3$; $\bar{Y} = 4$;
   $\sum(X - \bar{X})^2 = 32$; $\sum(Y - \bar{Y})^2 = 29$; $\sum(X - \bar{X})(Y - \bar{Y}) = 28$;
   The sample observation size is 10. Please calculate:
   (1)(5%)The regression line $\hat{Y} = \hat{\alpha} + \hat{\beta}X$
   (2)(5%)The variance of residuals $\hat{\sigma}^2$。
   (3)(5%)The t-statistic of the test $H_0: \beta = 0$。

6. (20%)Throwing two fair dice, each has six faces (1,2,3,4,5,6 points). $X_1$, $X_2$ represent the number of results respectively. Y=max($X_1$,$X_2$)。 Please calculate:
   (1) (10%)E(Y)。
   (2) (10%)Var(Y)。

7. (15%)There are N students in a class. How large N will let the probability of at least two students having the same birthday exceed 90%? Note: A year has 365 days.
   【Hint: You may use the following characters:
   (1) $e^{-x} \geq 1 - x$; (2) $e^{-1} = 0.3678$; $\ln(0.1) = -2.30259$
   (3) $m \times (m+1) \times (m+2) \ldots (m+k) \leq \left(m + \frac{k}{2}\right)^{k+1}$ 】

試題隨卷繳交