國立臺灣科技大學 112 學年度碩士班招生試題

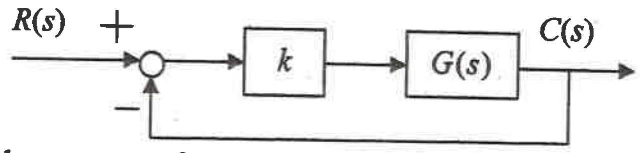
系所組別：機械工程系碩士班丁組

科　　目：系統控制

（總分為 100 分；所有試題務必於答案卷內頁依序作答，否則不予計分）

題目共八大題，總分 100 分，每小題有標示所占分數，請依序作答

Problem 1:(10%) Consider a unity feedback system shown below where $G(s)=\dfrac{1}{s(s+2)(s+10)}$ and k is the controller gain.



(a) Find the steady state error for a unit ramp input with $k=200$. (5%)
(b) Find the steady state error for a unit ramp input with $k=400$. (5%)

Problem 2: (10%) Definiteness of a scalar function is very important in the Lyapunov stability theory. Let $\mathbf{x}=[x_1,\ldots,x_n]^T$ be the state vector, and a scalar function $V(\mathbf{x}):\Re^n\rightarrow\Re$ is positive definite (p.d.) if it satisfies the conditions $V(\mathbf{0})=0$ and $V(\mathbf{x})>0$ for all $\mathbf{x}\neq\mathbf{0}$. It is positive semi-definite (p.s.d.) if $V(\mathbf{0})=0$ and $V(\mathbf{x})\geq 0$ for all $\mathbf{x}\neq\mathbf{0}$. Likewise, we may also have the definitions for negative definite (n.d.) functions as well as negative semi-definite (n.s.d.) functions. It is indefinite if not belongs to anyone above. Based on these definitions, please determine definiteness of the following scalar functions with clear reasoning

(a) $V(x_1,x_2)=x_1^2+x_2^2$ (2%)
(b) $V(x_1,x_2)=x_1^2-x_2^2$ (2%)
(c) $V(x_1,x_2)=-x_1^2$ (2%)
(d) $V(x_1,x_2)=(x_1+x_2)^2$ (2%)
(e) $V(x_1,x_2)=x_1^2+x_2^2+(x_1-x_2)^2$ (2%)

Problem 3: (12%) A state of a linear time-invariant (LTI) system is controllable, if the control u may drive it from anywhere in the state space to the origin; otherwise, the state is uncontrollable. An LTI system is completely controllable, if all states are controllable. An LTI system is stabilizable if all uncontrollable states are stable, i.e., we may construct a stabilizing controller to move all the controllable poles to the desired places, and we don't need to do anything to the uncontrollable poles because they are already stable. By inspection, please answer the questions briefly.

(a) Which ones of the following systems are completely controllable? (3%)
(b) Which ones are stabilizable? (3%)
(c) Which ones are stabilizable but not completely controllable? (3%)
(d) Which ones are with uncontrollable states? (3%)

System 1: $\begin{cases}\dot{x}_1=x_2\\ \dot{x}_2=u\end{cases}$　System 2: $\begin{cases}\dot{x}_1=x_1\\ \dot{x}_2=u\end{cases}$　System 3: $\begin{cases}\dot{x}_1=-x_1\\ \dot{x}_2=u\end{cases}$

System 4: $\begin{cases}\dot{x}_1=-x_2\\ \dot{x}_2=u\end{cases}$　System 5: $\begin{cases}\dot{x}_1=x_2\\ \dot{x}_2=-u\end{cases}$

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Problem 4: (10%) Please compute the value of $G(s)=\dfrac{10(s+1)^{20}}{s(s+9)^{20}}$ for $s=1+j$.

Hint 1: For complex numbers $s_i=\sigma_i+j\omega_i=|s_i|\angle\theta_i$ with $i=1,2$, we have the properties $s_1s_2=|s_1||s_2|\angle(\theta_1+\theta_2)$ and $\dfrac{s_1}{s_2}=\dfrac{|s_1|}{|s_2|}\angle(\theta_1-\theta_2)$.

Hint 2: You may use the approximations:

$$\left(\sqrt{5}\right)^{20}\approx 10^7,\ \left(\sqrt{101}\right)^{20}\approx 10^{20},\ \tan^{-1}\frac{1}{2}\approx 26^\circ,\ \tan^{-1}\frac{1}{10}\approx 6^\circ.$$

Problem 5: (10%) Consider a linear time-invariant control system

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ 2 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u$$

$$y=[1\quad 0]\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

(a) Determine the gains in the state feedback controller $u=-k_1x_1-k_2x_2$ so that the closed-loop poles are at -1 and -1. (5%)

(b) Suppose the states are not all available but the output y can be obtained by sensor feedback. An observer is constructed as

$$\begin{bmatrix}\dot{\hat{x}}_1\\ \dot{\hat{x}}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ 2 & 1\end{bmatrix}\begin{bmatrix}\hat{x}_1\\ \hat{x}_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u+\begin{bmatrix}l_1\\ l_2\end{bmatrix}(y-\hat{y})$$

$$\hat{y}=[1\quad 0]\begin{bmatrix}\hat{x}_1\\ \hat{x}_2\end{bmatrix}$$

Please find the observer gains l_1 and l_2 so that the closed-loop observer poles are at -10 and -10. This way the observer states may converge 10-times faster than the system states. (5%)

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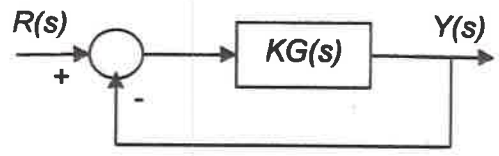
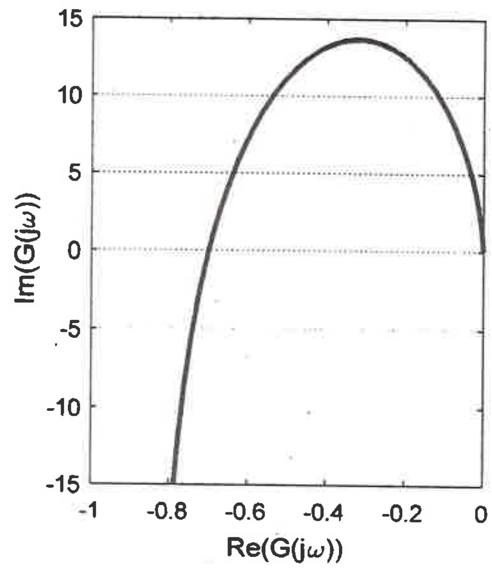
Problem 6: (15%) Consider the unity feedback structure (as shown below), and assume that the linear proper system G(s) does not have right half plane poles and zeros. If the Nyquist plot of G(jω) is as shown below, answer the followings:
(please note that the scales of the two axes are different, and the Nyquist plot beyond the extent of the plot has similar trend and therefore excluded.)

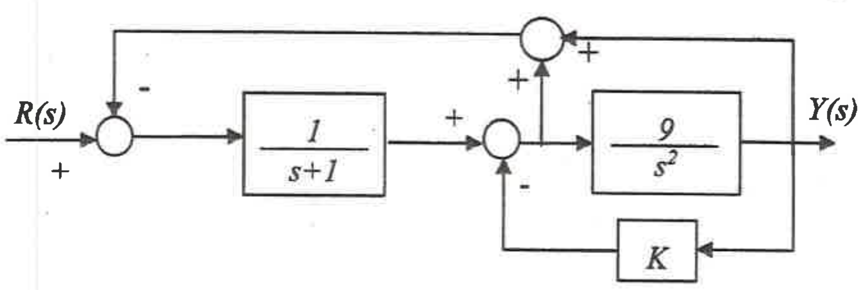
(a) What is the system type of G(s)? (3%)
(b) What is the number of asymptotes on the root locus of G(s)? (3%)
(c) What is the slope of the Bode magnitude plot of G(s) at high frequency range? (3%)
(d) What is the slope of the Bode magnitude plot of G(s) at low frequency range? (3%)
(e) Estimate the gain margin of the system. (3%)





Problem 7: (15%) For the system below,

(a) Use root locus technique to show the locations of the roots for the transfer function $\frac{Y}{R}(s)$ as K varies from 0 to infinity. (10%)
(b) Determine the range of K such that the closed loop system is stable. (5%)



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Problem 8: (18%) A pulley (of radius r) is used to hold an object of mass M near the specified height (x_0). Unfortunately, the pulley is heavy so the moment of inertia J is not negligible, and subjects to significant viscous friction on the pulley shaft (modelled by a viscous damping of coefficient D). Due to this fact, the rope-pulley contact my slip and the friction force between the rope and the pulley is modelled by another viscous damping coefficient f_v. Note that the gravitational force Mg will be balanced by an equilibrium force F_0, and we would like to investigate the dynamics of the system with small perturbation ΔF about F_0, i.e., $x=x_0+\Delta x$ and $F=F_0+\Delta F$.

(a) Write the governing equations of this system in the s-domain, and derive the transfer function $\frac{\Delta x}{\Delta F}(s)$. (10%)

(b) Supposed that the rope is replaced with the bungee jumping rope by mistake, and the spring effect of the rope cannot be ignored. Include the spring effect and **write the governing equations of the whole system,** define intermediate variables as needed. (8%)

(hint: use two springs to model the effect for the horizontal part and vertical part of the rope respectively)