國立臺灣大學 112 學年度碩士班招生考試試題

1. Let
$$A = \begin{pmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{pmatrix}.$$
where $x \in \mathbb{R}$. Determine the rank of A according to the value of x. (15 points)

2. Find
   (i) $\lim_{n\to\infty} \sum_{i=0}^{n-1} \left(\frac{i}{n}\right)^2 \times \frac{1}{n}$; (3 points)
   (ii) $\lim_{n\to\infty} \frac{n^k}{e^n}$; (5 points)
   (iii) $\lim_{n\to\infty} n^{1/n}$. (7 points)

3. Let
$$A = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}.$$
Find
   (i) all eigenvalues of A; (5 points)
   (ii) all eigenvectors of A; (5 points)
   (iii) a diagonal matrix D and an invertible matrix C such that $A = CDC^{-1}$. (5 points)

4. Compute the following derivative and integral
   (i) $\frac{\mathrm{d}}{\mathrm{d}x} \int_x^{x^3} \sqrt{t} \sin t \mathrm{d}t$; (5 points)
   (ii) $\int e^x \cos x \mathrm{d}x$. (10 points)

5. Let
$$A = \begin{pmatrix} 1 & 1 & 2 & 3 \\ 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & 2 \end{pmatrix}.$$
Find a basis for
   (i) the row space of A; (5 points)
   (ii) the column space of A; (5 points)
   (iii) the null space of A. (10 points)

6. Employ the technique of Lagrange multipliers to find the maximum and minimum of $f(x, y) = xy - x$ subject to the constraint $x^2 + y^2 = 1$. (20 points)

試題隨卷繳回