國立中正大學

111學年度碩士班招生考試

試題

[第1節]

科目名稱	工程數學
系所組別	化學工程學系

—作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始40分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

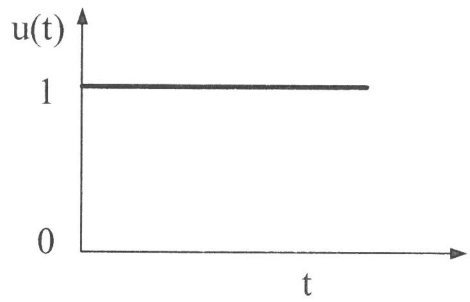
(1) **(15%)** Solve $(x^2+y^2+x)dx+xydy=0$

(a) Find an integrating factor if the equation is not exact. **(5%)** (b) Find the solution. **(5%)** (c) Determine the constant if $x=1$ and $y=0$. **(5%)**

(2) **(20%)** Using the Laplace transformation, solve the initial value problem

$$\frac{d^2y}{dt^2}+2\frac{dy}{dt}+y=u(t),\ y(0)=y'(0)=0$$

where the force input $u(t)$ is the unit step function as below:



(a) Take the Laplace transform of the differential equation. **(5%)** (b) Determine the characteristic equation and the eigenvalues of the problem. **(5%)** (c) Find the solution of $y(t)$. **(5%)** (d) Determine the steady-state solution $y(\infty)$. **(5%)**

(3) **(15%)** Find the solution $y(t)$ of the two-point boundary value problem

$$\frac{d^2y}{dt^2}-25y=25t,\ y(0)=1,\ y'(1)=0$$

(a) Find the homogeneous solution. **(5%)** (b) Determine the particular solution. **(5%)** (c) Determine constants of the solution. **(5%)**

(4) **(10%)** For any two n × n-matrices **A** and **C**, prove

(a) $(\mathbf{AC})^{\mathrm{T}}=\mathbf{C}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}$ **(5%)**

(b) $(\mathbf{AC})^{-1}=\mathbf{C}^{-1}\mathbf{A}^{-1}$ **(5%)**

(5) **(10%)** For any two vectors **u** and **v**, prove

div ($\mathbf{u}\times\mathbf{v}$) = $\mathbf{v}\bullet$curl $\mathbf{u}$ - $\mathbf{u}\bullet$curl $\mathbf{v}$

(6) **(30%)** Consider the case in which a linear string of length L, fastened at its two ends and is released at time zero from its horizontal stretched position (zero initial vertical displacement) but with a nonzero initial velocity. At time t, $y(x, t)$ is the vertical displacement of the particle of string having coordinate x. The initial vertical velocity of string is $g(x)$. The boundry value problem modeling this phenomenon is

$$\frac{\partial^2 y}{\partial t^2}=c^2\frac{\partial^2 y}{\partial x^2} \qquad 0<x<L,\ t>0,\text{ where } c^2 \text{ is a constant.}$$

(a) Write down the boundary conditions **(5%)** and (b) initial conditions of the problem. **(5%)**

(c) Find the solution of $y(x, t)$. **(20%)**