國立臺北大學 111 學年度碩士班一般入學考試試題

系（所）組別：金融與合作經營學系

科　　　目：統計學

☑可 □不可使用計算機

1、Let Y= $\beta_1 + \beta_2 X + \varepsilon$, where ε is the random error term with mean of 0 and variance of σ^2. Moreover, the mean-centered variables of (Y, X) are as follows:

i	1	2	3	4	5
$Y-\bar{Y}$	-2.3	-0.7	0	1.3	1.7
$X-\bar{X}$	-2	-1	0	1	2

(1) (5%) What is b_2, the OLS estimate of β_2?
(2) (3%) What is the estimate of σ^2?
(3) (2%) What is the estimate of Var(b_2)?

2、Let X and Y have the joint p.d.f. described as follows:

(x, y)	(1, 1)	(1, 2)	(1, 3)	(2, 1)	(2, 2)	(2, 3)
f(x,y)	2/15	4/15	3/15	1/15	1/15	4/15

and f(x,y) is equal to zero elsewhere.

(1) (10%) Find the means μ_x and μ_y, the variances σ_x^2 and σ_y^2, and the correlation coefficient ρ.
(2) (6%) Find the conditional p.d.f. f(y|X=1) and f(y|X=2).
(3) (4%) Compute E(Y|X=1), E(Y|X=2).

3、Let Z_n be a random variable with $\chi^2(n)$ and let $W_n = Z_n/n^2$.
(1) (5%) Find the moment generating function of W_n.
(2) (5%) Find the limiting distribution of W_n.

4、Let $f(x; \theta) = (1/\theta)x^{(1-\theta)/\theta}$, $0 < x < 1$, $0 < \theta < \infty$. Let $X_1, X_2, \ldots, X_n$ denote a random sample of size n from this distribution.
(1) (5%) Show that the maximum likelihood estimator of θ is $\hat{\theta} = -(\frac{1}{n})\sum_{i=1}^{n}\ln(X_i)$.
(2) (5%) Show $E(\hat{\theta}) = \theta$ and thus $\hat{\theta}$ is an unbiased estimator of θ.

5、(12%) An opposition party asserts that there being 64% citizens do not support a new policy launched by the government. If their assertion is near true, how many numbers of persons have to be surveyed if they want the estimation error less than 0.04 under confidence level 95%? On the other hand, if the real ratio is not known, how many numbers of persons have to be surveyed?【$Z_{0.975}$=1.96，$Z_{0.95}$=1.645】

6、(12 %) A random variable Y follows Uniform(-1,3). Derive the probability density function of W=Y^2.

7、(12 %) A test has probability q of success. X represents the random variable of trial times that first success happens. Please derive E(X)= 1／q.

8、(14 %) P represents the probability of adverse reaction not happening for a vaccination. P is a random variable with density functions:

$$\begin{cases} 12p^2(1-p) & 0 \le p \le 1 \\ 0 & otherwise \end{cases}$$

Let n be the number of persons who have been injected. Y is the number of persons that adverse reaction does not happen after vaccination. Please calculate E(Y) if n=10.

試題隨卷繳交