國立臺北大學 111 學年度碩士班一般入學考試試題

系（所）組別：財政學系

科　　目：微積分

☐可 ☑不可 使用計算機

1. (10 %) Find the limits: (a) $\lim_{x\to\infty} \frac{3x^5}{e^{7x}}$, (b) $\lim_{x\to\infty} \frac{(\ln x)^{12}}{x^6}$.

2. (14 %) Compute the second-order derivatives of each of the following functions:

   (a) $\log_{10} x$, (b) $\frac{x}{\ln x}$.

3. (14 %) For each of the questions, determine the area of the region bounded by the given set of curves.

   (a) $x = y^2 + 1,\ x = 0,\ y = -1,\ y = 2$

   (b) $y = \frac{1}{2}x^3 + 2,\ y = x + 1,\ x = 0,\ x = 2$

4. (10 %) Find the Taylor Series for $f(x) = 1/x^2$ about $x = -1$.

5. (10 %) Find dy/dx by implicit differentiation: $x \cdot \sin y = y \cdot \cos x$.

6. (12 %) Suppose that a line in the plane passes through point (0, 4). The slope of this line is m. Let $d(m)$ be the distance from point (3, 1) to the line.

   (a) Compute the function $d(m)$.

   (b) Find a value of m at which $d(m)$ is NOT differentiable.

7. (10 %) Suppose that n is an arbitrary positive integer. Use the integration by parts formula to show that $\gamma(n) = (n-1)!$, where

   $$\gamma(n) = \int_0^\infty x^{n-1} e^{-x}\, dx \ .$$

8. (10 %) Find dy/dx where $y = \int_x^{x+3} z(5-z)dz$.

9. (10 %) Evaluate the integral: $\int_0^2 \int_x^2 x\sqrt{1+y^3}\, dydx$

試題隨卷繳交