考 試 科 目	統計學	系 所 別	財務管理學系碩士班	考 試 時 間	2 月 10 日（四 ）第四 節

注意：「本科可使用一般計算機，但不得攜帶且不得使用具儲存程式功能之計算機」

1. (15%) Suppose we have three boxes, the first of which has 1 red ball and 9 white balls, the second of which has 5 red balls and 5 white balls, and the third of which has 9 red balls and 1 white ball. A box is chosen at random and a ball is sampled. Given that the observed ball is red, what is the probability that the third box was chosen? (Hint: 機率以分數表示)

2. (20%) The share price of some stock company varies in a random manner, such that the price increase each minute is described by a discrete random variable X (measured in dollars), with the following probability mass:

$$f_X(x) = \begin{cases} 0.5, & x = +0.05 \\ 0.2, & x = 0.00 \\ 0.3, & x = -0.05 \end{cases}$$

   a. (10%) Calculate $E(X) = \mu$ and $var(X) = \sigma^2$.
   b. (10%) Use the Central Limit Theorem to estimate the probability that the price will increase by 1.20 dollars, or more, after 3 hours.

3. (15%) A financial officer for a company wants to estimate the percent of accounts receivable that are more than 30 days overdue. He surveys 500 accounts and finds that 300 are more than 30 days overdue. Compute a 90% confidence interval for the true percent of accounts receivable that are more than 30 days overdue, and interpret the confidence interval.

4. (15%) The following data provides a recent survey of the youngest online entrepreneurs whose net worth is estimated at one million US dollars or more. Their ages range from 17 to 30. The third column of the table illustrates the number of entrepreneurs who correspond to the specific age group and their net worth. Are the ages and net worth independent? Perform a test of independence at the 5% significance level. (Hint: State the null hypothesis, expected values for the number of entrepreneurs, test statistics, decision rule and conclusion.)

Age Group	Net Worth Value	Number of Entrepreneurs
17-25	1-5	8
26-30	1-5	6
17-25	6-24	7
26-30	6-24	5
17-25	≥ 25	5
26-30	≥ 25	9

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5. (20%) A consumer research group is interested in testing an automobile manufacturer's claim that a new economy model will travel at least 25 miles per gallon of gasoline (H_0: $\mu \geq 25$).

   a. (10%) With a .02 level of significance and a sample of 30 cars, what is the decision rule based on the value of $\bar{x}$ for the test to determine whether the manufacturer's claim should be rejected? Assume that σ is 3 miles per gallon.

   b. (10%) What is the probability of committing a Type II error if the actual mileage is 23 miles per gallon?

6. (15%) A sales manager collected the following data on annual sales (y) for new customer accounts and the number of years of experience (x) for a sample of 10 salespersons. An estimated regression equation is $\hat{y} = 80 + 4x$. Compute the coefficient of determination R^2 and interpret.

Salesperson	Years of Experience (x)	Annual Sales ($1000s) ($y$)
1	1	80
2	3	97
3	4	92
4	4	102
5	6	103
6	8	111
7	10	119
8	10	123
9	11	117
10	13	136

Note:

- The CDF of the standard normal distribution is denoted by the Φ function:
  $\Phi(x) = P(Z \leq x)$. $\Phi(1.025) = 0.8473$, $\Phi(1.15) = 0.8749$, $\Phi(1.20) = 0.8849$, $\Phi(1.22) = 0.8888$, $\Phi(1.282) = 0.90$, $\Phi(1.61) = 0.9463$, $\Phi(1.645) = 0.95$, $\Phi(1.82) = 0.9656$, $\Phi(1.96) = 0.975$, $\Phi(2.051) = 0.9798$.

- Chi-Square Distribution Table: χ^2 critical value with 10 degrees of freedom and a 0.01 probability in the upper tail is $\chi^2_{0.01,10} = 23.209$. $\chi^2_{0.025,2} = 7.3778$, $\chi^2_{0.01,2} = 9.2103$, $\chi^2_{0.05,2} = 5.9914$, $\chi^2_{0.025,3} = 9.3484$, $\chi^2_{0.01,3} = 11.3449$, $\chi^2_{0.05,3} = 7.8147$.

備　　註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。