Instructions:

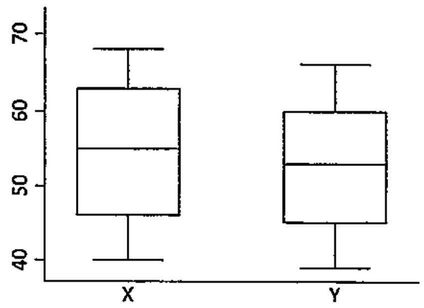
- Write in Chinese or English only.
- Make sure all your answers are legible, unquestionably labeled, and clearly explained (with equations if possible).
- A standard normal probability table is attached on the last page.

1. A street vendor sells apples and oranges. The number of apples she sells each day can be described by X and the number of oranges she sells each day can be described by Y. The vendor earns \$0.4 from each apple and \$1.1 from each orange. Suppose that $\mathrm{E}(X) = 80, \mathrm{E}(Y) = 50, \mathrm{Var}(X) = 49, \mathrm{Var}(Y) = 25$, and $\mathrm{Cov}(X, Y) = -10$.
   (a) **(10 points)** Calculate the vendor's expected daily profits and the standard deviation in the vendor's daily profits.
   (b) **(10 points)** Suppose the numbers of the two products sold each day are independent of previous days. Calculate the vendor's expected weekly (7 days) profits and the standard deviation in the vendor's weekly (7 days) profits.

2. **(10 points)** A mutual fund manager is considering investments in two stocks, A and B. The percentage return on A is normally distributed with mean .18 and standard deviation .10. The percentage return on B is normally distributed with mean .25 and standard deviation .20. The returns on the two stocks are independent of each other. Suppose the manager invests \$2 million in A and \$3 million in B. Calculate the probability that she loses money.

3. A travel agency hopes a training program to make its agents more productive. Eighty randomly selected agents are assigned to the training program, and another eighty randomly selected agents are assigned to the control group. Data are collected on the number of clients each serves in the training group $(X_1, X_2, \ldots, X_{80})$ and in the control group $(Y_1, Y_2, \ldots, Y_{80})$ during one week a month after the program is over. Below are summary statistics on the number of clients served during the week along with box plots showing the distribution of data by group.

	X_i	Y_i	$D_i = X_i - Y_i$
N	80	80	80
Mean	54.5	52.7	1.8
Std. dev.	8.7	7.7	11.3
Min.	40	39	-24
Max.	68	66	25



   (a) **(4 points)** What are the hypotheses for evaluating whether the average number of clients served by an agent is different between the two groups?
   (b) **(6 points)** Derive the sampling distribution of estimator for the parameter of interest.
   (c) **(4 points)** Calculate a test statistic based on your sampling distribution from Part (b).
   (d) **(2 points)** Make a conclusion of the hypothesis test at the 10% level of significance.
   (e) **(4 points)** What type of error might have been made in the conclusion of your test?

4. Suppose that the dependent variable, y, is related to the independent variable, x, and the error (or disturbance), u, as

$$y = \beta_0 + \beta_1 x + u. \tag{1}$$

We have a *i.i.d.* random sample of size n, $\{(x_i, y_i) : i = 1, 2, \ldots, n\}$.

(a) **(3 points)** Determine $\widehat{\beta}_{0,OLS}$ and $\widehat{\beta}_{1,OLS}$, the ordinary least squares (OLS) estimator for β_0 and β_1. What condition do you need to obtain $\widehat{\beta}_{0,OLS}$ and $\widehat{\beta}_{1,OLS}$?

(b) **(3 points)** Suppose that the error u has an expected value of zero given any value of the explanatory variable, *i.e.*,

$$\mathbb{E}(u|x) = 0.$$

Show that $\widehat{\beta}_{1,OLS}$ is an unbiased estimator for β_1, *i.e.*,

$$\mathbb{E}\left(\widehat{\beta}_{1,OLS}\right) = \beta_1.$$

(c) **(3 points)** Following b., show that $\widehat{\beta}_{1,OLS}$ is a consistent estimator for β_1, *i.e.*, as $n \to \infty$,

$$\widehat{\beta}_{1,OLS} \xrightarrow{p} \beta_1.$$

(d) **(3 points)** Suppose that the error u has the same variance given any value of the explanatory variable, *i.e.*,

$$\mathbb{E}\left(u^2|x\right) = \sigma^2.$$

Determine the conditional variance of $\widehat{\beta}_{1,OLS}$ given $\{x_i : i = 1, 2, \ldots, n\}$.

(e) **(3 points)** Suppose that $u_i \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma^2)$. Determine $\widehat{\beta}_{0,MLE}$, $\widehat{\beta}_{1,MLE}$, and $\widehat{\sigma}^2_{MLE}$, the maximum likelihood estimator for β_0, β_1, and σ^2.

(f) **(2 points)** Suppose that equation (1) is the true data generating process. However, the econometrician would like to estimate another model:

$$y = bx + v. \tag{2}$$

Determine $\widehat{b}_{OLS}$, the OLS estimator for b.

(g) **(3 points)** Determine $\mathbb{E}\left(\widehat{b}_{OLS}\right)$, the expectation of $\widehat{b}_{OLS}$, in terms of β_0, β_1, and $\{x_i : i = 1, 2, \ldots, n\}$.

5. Consider the following data

	y	x
1	32	8
2	50	10
3	8	4
4	2	2
5	18	6

(a) **(2 points)** Estimate the relationship between y and x using the ordinary least squares (OLS); that is, obtain the intercept and slope estimates in the equation

$$y_i = \beta_0 + \beta_1 x_i + u_i, \quad i = 1, 2, \ldots, 5.$$

(b) **(2 points)** Compute the fitted values and residuals for each observation.

(c) **(2 points)** Compute the coefficient of determination, *i.e.*, the R^2.

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6. Suppose the data generating process is:

$$y_i = \beta_0 + \beta_1 x_i + u_i, \quad i = 1, 2, \ldots, n, \tag{3}$$

where $\{(x_i, u_i) : i = 1, 2, \ldots, n\}$ are *i.i.d.* with $\mathbb{E}(u_i|x_i) = 0$. Consider the reverse regression estimator. That is, consider the following model:

$$x_i = \theta_0 + \theta_1 y_i + v_i. \tag{4}$$

Let $\widehat{\theta}_{0,OLS}$ and $\widehat{\theta}_{1,OLS}$ be the ordinary least squares estimator for θ_0 and θ_1, $\widehat{x}_i = \widehat{\theta}_{0,OLS} + \widehat{\theta}_{1,OLS} \cdot y_i$, and $\widehat{v}_i = x_i - \widehat{x}_i$.

(a) **(4 points)** Which, if any, of the following hold and why:

i. $\sum_{i=1}^{n} \widehat{v}_i = 0$.
ii. $\sum_{i=1}^{n} x_i \widehat{v}_i = 0$.
iii. $\sum_{i=1}^{n} y_i \widehat{v}_i = 0$.

(b) **(4 points)** Is $\widehat{\theta}_{1,OLS}$ an unbiased estimator for $1/\beta_1$?

(c) **(4 points)** Is $\widehat{\theta}_{1,OLS}$ a consistent estimator for $1/\beta_1$?

7. Consider the wage equation for married, working women:

$$\ln(wage_i) = \beta_0 + \beta_1 exper_i + \beta_2 exper_i^2 + \beta_3 educ_i + \beta_4 age_i + \beta_5 kidslt6_i + \beta_6 kidsge6_i + u_i,$$

where $exper_i$, $educ_i$, age_i, $kidslt6_i$, and $kidsge6_i$ are respectively woman i's working experience (years), education (years), age (years), number of children less than six, and number of children at least six years of age.

Using the data on the 428 working, married women from Mroz, 1987, we obtain the following estimation results:

	Estimate	Std. Error	t value	Pr(>|t|)
Intercept	-0.4209	0.3169	-1.328	0.185
exper	0.0398	0.0134	2.973	0.003
expersq	-0.0008	0.0004	-1.942	0.053
educ	0.1078	0.0144	7.487	0.000
age	-0.0015	0.0053	-0.277	0.782
kidslt6	-0.0607	0.0888	-0.684	0.494
kidsge6	-0.0146	0.0279	-0.523	0.601

$n = 428$. $R^2 = 0.158$.

(a) **(4 points)** Can you conclude that, after the productivity variables (`exper`, `expersq`, and `educ`) are controlled for, women are *not* paid differently depending on their age and number of children (`kidslt6` and `kidsge6`)? Why or why not?

(b) **(4 points)** Some argue that in the wage equation, at least one very important variable is missing: the *adaptive skill*, *i.e.*, the skill to cope in the environment with greatest success and least conflict with others. However, the adaptive skill is *unobserved*. Suppose that the adaptive skill is uncorrelated to all the variables in our equation. Should we worried about the omitted-variable bias for the estimation for the coefficients of age, `kidslt6`, and `kidsge6`? Why or why not?

(c) **(4 points)** To test the omitted-variable bias, one suggests to test the exogeneity condition, *i.e.*, to test if the error-terms are uncorrelated to the independent variables. Since the error-terms are not observable, one suggests to calculate the residuals, and to calculate the correlation coefficient of the residuals and the independent variables as the test statistics. Do you agree? Why or why not?

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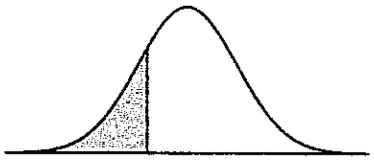
Standard normal probability table



negative Z

Second decimal place of Z										
0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.00	Z
0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	−3.4
0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005	−3.3
0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007	−3.2
0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010	−3.1
0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013	−3.0
0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019	−2.9
0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026	−2.8
0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035	−2.7
0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047	−2.6
0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062	−2.5
0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082	−2.4
0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107	−2.3
0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139	−2.2
0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179	−2.1
0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228	−2.0
0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287	−1.9
0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359	−1.8
0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446	−1.7
0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548	−1.6
0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668	−1.5
0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808	−1.4
0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968	−1.3
0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151	−1.2
0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357	−1.1
0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587	−1.0
0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841	−0.9
0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119	−0.8
0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420	−0.7
0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743	−0.6
0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085	−0.5
0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446	−0.4
0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821	−0.3
0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207	−0.2
0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602	−0.1
0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000	−0.0

*For $Z \leq -3.50$, the probability is less than or equal to 0.0002.

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