國立臺灣師範大學 109 學年度**碩士班**招生考試試題

科目：**統計學** 適用系所：**管理研究所**

注意：1.本試題共 2 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則依規定扣分。

1. A package, say A, of 24 crocus bulbs contains 8 yellow, 8 white, and 8 purple crocus bulbs. A package, say B, of 24 crocus bulbs contains 6 yellow, 6 white, and 12 purple crocus bulbs. One of the two bags is selected at random.
   (a) If 3 bulbs from this bag were planted and all 3 yielded purple flowers, compute the conditional probability that package B was selected. ***(11 points)***
   (b) If the 3 bulbs yielded 1 yellow flower, 1 white flower, and 1 purple flower, compute the conditional probability that package A was selected. ***(11 points)***

2. Let **X** and **Y** equal the concentration in parts per billion of chromium in the blood for healthy persons and for persons with a suspected disease, respectively. Assume that the distributions of **X** and Y are $N(\mu_1,\sigma_1^2)$ and $N(\mu_2,\sigma_2^2)$, respectively. Using $n=8$ observations of **X**:

   15, 23, 12, 18, 9, 28, 11, 10

   and $m=10$ observations of **Y**:

   25, 20, 35, 15, 40, 16, 10, 22, 18, 32

   (a) Give a point estimate of σ_1^2/σ_2^2. ***(7 points)***
   (b) Find a 95% confidence interval for σ_1^2/σ_2^2. ***(7 points)***
   (c) Give a point estimate of $\mu_1 - \mu_2$. ***(7 points)***
   (d) Find a 95% confidence interval for $\mu_1 - \mu_2$. ***(7 points)***

Note: $t_{(0.025,\ 16)}=2.12$, $t_{(0.025,\ 18)}=2.101$, $F_{(0.025,7,9)}=4.2$, $F_{(0.025,9,\ 7)}=4.82$.

3. Students' scores on the mathematics portion of the ACT examination, X, and on the final examination in the first semester calculus (200 points possible), Y, are given as follows:

x	25	20	26	26	28	28	29	32	20	25	26	28	25	31	30
y	138	84	104	112	88	132	90	183	100	143	141	161	124	118	168

   (a) Calculate the least squares regression line for these data. ***(6 points)***
   (b) Test H_0: $\beta=0$ against H_1: $\beta>0$ at the $\alpha=0.025$ significance level using a t-test. ***(6 points)***
   (c) Test H_0: $\beta=0$ against H_1: $\beta\neq0$ at the $\alpha=0.05$ significance level by setting up the ANOVA table and using an F test. ***(6 points)***
   (d) Find a 95% confidence interval for $\mu(x)$ when $x=20$, 25, and 30. ***(6 points)***

(e) Find a 95% prediction interval for Y when x=20, 25, and 30. ***(6 points)***

Note: $t_{(0.025,\ 13)}=2.16$, $t_{(0.025,\ 14)}=2.145$, $t_{(0.05,\ 13)}=1.71$, $t_{(0.05,\ 14)}=1.761$
$F_{(0.025,1,13)}=6.414$, $F_{(0.025,1,\ 14)}=6.298$, $F_{(0.05,1,13)}=4.667$, $F_{(0.05,1,\ 14)}=4.6$

4. Let $X_1, X_2, \ldots, X_n$ be a random sample from the distribution with *probability density function* $f(x; p)=p(1-p)^{x-1}$, x=1, 2, 3, …, where $0<p<1$.
   (a) Show that $Y=\sum_{i=1}^{n} X_i$ is a sufficient statistic for p. ***(5 points)***
   (b) Find a function of $Y=\sum_{i=1}^{n} X_i$ that is an unbiased estimator of $\theta = 1/p$. ***(5 points)***
   (c) Find the Cramer-Rao lower bound for the variance of the above unbiased estimator. ***(5 points)***
   (d) Find the maximum likelihood estimator for $\theta = 1/p$. ***(5 points)***