國立高雄師範大學 108 學年度碩士班招生考試試題

系所別：數學系

科　目：線性代數

※注意：1.作答時請將試題題號及答案依序寫在答案卷上，於本試題上作答者，不予計分。
2.答案卷限用藍、黑色筆作答，以其他顏色作答之部分，該題不予計分。

1. (12%) Let V be a vector space with the zero vector θ over a field F and $v \in V$. Show that the set $\{v\}$ is linearly independent **if and only if** $v \neq \theta$.
2. (10%) Let A be a singular matrix. Prove or disprove: adj(A) is singular.
3. Let

$$H = \begin{bmatrix} 0 & 1 & -2 & 1 \\ 5 & 0 & 0 & 7 \\ 0 & 1 & -1 & 0 \\ 3 & 0 & 0 & 2 \end{bmatrix} \text{ and } K = \begin{bmatrix} 2 & 1 & 2 & 1 \\ 3 & 0 & 1 & 1 \\ -1 & 2 & -2 & 1 \\ 0 & 2 & 0 & 1 \end{bmatrix}.$$

   (a) (10%) Find H^{-1}.
   (b) (8%) Find tr($K^{-1}HK^2$).
   (c) (10%) Let $G = HK$. Find det(adj(G)).

4. (20%) Let $D = P^{-1}AP$, where $D = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$ and $P = \begin{bmatrix} 1 & 1 & 3 & 2 \\ -2 & 0 & 7 & 8 \\ 1 & 0 & -3 & -4 \\ 1 & 1 & 2 & 3 \end{bmatrix}$.

   (a) What are the eigenvalues of A?
   (b) Give the eigenspaces for each eigenvalue in part (a).
   (c) What is $\det(A)$?

（背面尚有試題）

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5. (15%) Consider $L: P_3 \to M_{22}$ given by $L(ax^3+bx^2+cx+d)=\begin{bmatrix} a-d & 2b \\ b & c+d \end{bmatrix}$.
   (a) Determine whether L is one-to-one and whether L is onto.
   (b) What is $\dim(\ker(L))$? What is $\dim(\mathrm{range}(L))$?

6. (15%) Determine whether the following statement is true or false. Explain why clearly and briefly.
   (a) Every orthogonal matrix is nonsingular.
   (b) If W is a nontrivial subspace of R^n, and $L: R^n \to R^n$ is given by $L(v) = proj_W v$, then the matrix for L with respect to the standard basis is an orthogonal matrix.
   (c) If v_1 and v_2 are eigenvectors corresponding to two distinct eigenvalues of a symmetric matrix A, then $v_1 \cdot v_2 = 0$.