

國立高雄師範大學 108 學年度碩士班招生考試試題

系所別：數學系

科 目：高等微積分

※注意：1.作答時請將試題題號及答案依序寫在答案卷上，於本試題上作答者，不予計分。

2.答案卷限用藍、黑色筆作答，以其他顏色作答之部分，該題不予計分。

1. (10%)

Show that $|\sin^2 x - \sin^2 y| \leq |x - y|$ for all $x, y \in \mathbb{R}$.

2. (a) (10%)

Let $f, g : [a, b] \rightarrow \mathbb{R}$ be bounded and E be a finite subset of $[a, b]$. Suppose that $f(x) = g(x)$ for all $x \in [a, b] \setminus E$ and f is integrable on $[a, b]$. Prove that g is integrable on $[a, b]$ and $\int_a^b g(x) dx = \int_a^b f(x) dx$.

(b) (10%)

If E is not finite, does it follow that g is integrable on $[a, b]$ and

$\int_a^b g(x) dx = \int_a^b f(x) dx$? Please show your answer.

3. (10%)

Let $f(x) = \int_x^{x+1} \sin e^t dt$. Prove that $e^x |f(x)| < 2$.

4. (10%)

Prove that the series $\sum_{k=1}^{\infty} \frac{1}{k} \sin \frac{x}{k+1}$ converges uniformly on any bounded subset E of \mathbb{R} .

(背面尚有試題)

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5. (15%)

Prove that the sequence $\{x_n\}_{n=1}^{\infty}$ defined by $x_n = \int_1^n \frac{\cos t}{t^2} dt$ is convergent.

(Hint: Every Cauchy sequence in R is convergent.)

6. (15%)

Consider a function given below

$$f(x) = \begin{cases} 1, & \text{if } x \in Q \\ 0, & \text{if } x \notin Q \end{cases}$$

Given any $c \in R$, prove that f is discontinuous at c .

7. (20%)

Let f be Riemann-integrable on $[a, b]$ satisfying $\int_a^b f^2(x) dx = 0$.

Suppose that f is continuous at x_0 . Prove $f(x_0) = 0$.