逢甲大學109學年度碩士班考試入學試題

編號：14　科目代碼：111

科目	工程數學	適用系所	自動控制工程學系	時間	90分鐘

※請務必在答案卷作答區內作答。　

1. For matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 2 \\ 0 & 2 & 1 \end{bmatrix}$,

(1) Determine the eigenvalues of matrix A. (5%)

(2) Determine the corresponding eigenvectors of matrix A. (15%)

2. The Fourier series of a function f defined on $(-p, p)$ is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cdot \cos \frac{n\pi}{p} x + b_n \cdot \sin \frac{n\pi}{p} x \right).$$

(1) Find the Fourier series of the following function:

$$f(x) = \begin{cases} 0, -\pi < x < 0 \\ x^2, 0 \le x < \pi \end{cases}. \text{ (15\%)}$$

(2) Use the result derived from (1) to show

$$\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots\cdots. \text{ (15\%)}$$

3. (20%) Find the inverse Laplace transform of the following functions:

(a) $\dfrac{e^{-4s}}{s^5}$, (b) $\dfrac{3s+4}{s^2+4s+7}$.

4. (10%) Write down the convolution theorem for Laplace transforms.

5. (20%) Solve the differential equation: $y''+y'+3y = \sin 2x$.