國立中正大學
109學年度碩士班招生考試
試 題

[第1節]

科目名稱	工程數學
系所組別	化學工程學系

－作答注意事項－

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始40分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

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系所組別：化學工程學系

1. Solve $(x-y^2)dx+2xydy=0$

   (a) The equation is exact or not. (b) Find an integrating factor if the equation is not exact. (c) Find the solution. (d) Determine the constant if $x=1$ and $y=1$. **(15%)**

2. Using the Laplace transformation, solve the initial value problem

$$\frac{dx}{dt}=2x-3y$$
$$\frac{dy}{dt}=y-2x$$
$$x(0)=8, y(0)=3$$

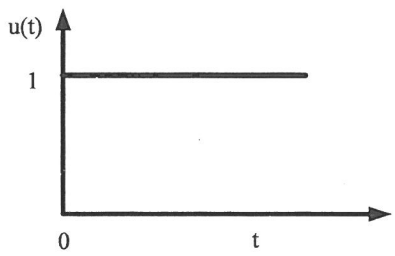
   (a) What are the characteristic equation and the eigenvalues of the problem?

   (b) Find the solution of $x(t)$ and $y(t)$.

   (c) Explain the solution is stable or not. **(15%)**

3. (a) Determine constants, a, b and c, so that the function $y_0(t)$ = a and $y_1(t)$ = b + c t form an orthonormal set on the interval $0 \le t \le 1$. **(10%)**

   (b) Find the Laplace transform F(s) of the function f(t) = u(t) + 2t + 3t^2, where u(t) is the unit step function as below: **(10%)**



4. Prove that the inverse of a nonsingular n × n matrix A = [a_{jk}] is given by

$$\mathbf{A}^{-1}=\frac{1}{\det \mathbf{A}}\left[A_{jk}\right]^T$$

   where det A is the determinant of A and A_{jk} is the cofactor of a_{jk} in det A. **(10%)**

5. Prove $\nabla\cdot(\mathbf{u}\times\mathbf{v}) = \mathbf{v}\cdot(\nabla\times\mathbf{u})-\mathbf{u}\cdot(\nabla\times\mathbf{v})$ **(10%)**

6. The one-dimensional heat equation is

$$\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}$$

   The two boundary conditions are $u(0, t) = 0$ and $u(L, t) = 0$ for all t.

   The initial condition is $u(x, 0) = f(x)$. Solve the above partial differential equation. **(30%)**