

# 國立臺北大學 109 學年度碩士班一般入學考試試題

系(所)組別：統計學系  
 科目：統計學

第1頁 共5頁  
可 不可使用計算機

一、(50%)

- (10%) Identical twins (同卵雙胞胎) form when a single fertilized egg splits into two genetically identical parts. The identical twins share the same DNA set and are thus always of the same sex. Fraternal (異卵雙胞胎) twins develop when two separate eggs are fertilized and implant in the uterus. The genetic connection is the same as siblings (兄弟姐妹) born at separate times. If 64 percent of all twin pairs are of the same sex, what percentage of twin pairs are identical twins?
- (10%, 5% each) We are interested in the outcome for rolling dice.
  - Find the sampling distribution when a pair of fair dice is rolled (mean and standard deviation)
  - Determine the variance of the sum obtained when a pair of fair dice is rolled.
- (10%) The mean of a Poisson random variable  $X$  is  $\mu = 4$  with standard deviation  $\sigma$ .  
 Find  $P(\mu - \sigma < X < \mu + \sigma)$ .
- (10%, 5% each) If  $X$  is a binomial random variable with  $B(100, 0.1)$ ,
  - Find  $P(12 \leq X \leq 14)$ .
  - Find the approximate value of  $P(12 \leq X \leq 14)$  using the normal approximation.
- (10%, 5% each) Let  $\bar{X}$ ,  $\bar{Y}$ ,  $S_X^2$  and  $S_Y^2$  be the respective sample means and unbiased estimates of the variances obtained from independent samples of sizes  $n$  and  $m$  from the normal distributions  $N(\mu_X, \sigma_X^2)$  and  $N(\mu_Y, \sigma_Y^2)$ , where  $\mu_X, \mu_Y, \sigma_X^2, \sigma_Y^2$  are unknown. If the sample sizes  $n$  and  $m$  are smaller than 30, construct  $(1 - \alpha)100\%$  confidence interval for  $\mu_X - \mu_Y$  when
  - two population standard deviations can be assumed equal.
  - two population standard deviations can not be assumed equal.

二、(50%)

- (15%) Employees of MNM Corporation are about to undergo a retraining program. Management is trying to determine which of three programs is the best. They believe that the effectiveness of the programs may be influenced by sex. A factorial experiment was designed. You are given the following information.

Factor A: Program	Factor B: Sex		Total( $y_{i..}$ )
	Male	Female	
Program I	320	380	1240
	240 (560)	300 (680)	
Program II	160	240	790
	180 (340)	210 (450)	
Program III	240	360	1270
	290 (530)	380 (740)	
<b>Total(<math>y_{.j.}</math>)</b>	1430	1870	3300

Calculations:  $\sum \sum \sum y_{ijk}^2 = 969800$ ;  $\sum y_{i..}^2 = 3,774,600$ ;  $\sum y_{.j.}^2 = 5,541,800$ ;  $\sum y_{...k}^2 = 1,922,600$ ;  $SST = 62,300$

試題隨卷繳交

接背面

# 國立臺北大學 109 學年度碩士班一般入學考試試題

系(所)組別：統計學系

科目：統計學

第2頁 共5頁

可 不可使用計算機

- State the ANOVA model for the experimental design (i.e. full model of the two-factor analysis of variance).
- Fit the ANOVA model (i.e. estimate the model parameters).
- Set up the ANOVA table and test for the significances of factor A, factor B, and interaction of factors A and B at the 5% level of significance.

2. (10%) A new brand of chocolate bar is being market tested. Four hundred of the new chocolate bars were given to consumers to taste. The consumers were asked whether they liked or disliked the chocolate bar. You are given their responses below.

Response	Frequency
Liked	320
Disliked	<u>80</u>
	400

- Construct a 90% confidence interval for the true proportion of people who liked the chocolate bar.
- With a 0.90 probability, how large of a sample needs to be taken to provide a margin of error of 3% or less?

3. (10%) The average starting salary of students who graduated from colleges of Business in 2018 was \$38,400. A sample of 100 graduates of 2019 showed an average starting salary of \$40,000. Assume the standard deviation of the **population** is known to be \$8000. We want to determine whether or not there has been a significant **increase** in the starting salaries.

- State the null and alternative hypotheses. Test the hypothesis at the 5% level of significance. What do you conclude?
- Compute the power of the statistical test when  $\mu = 40,000$ ?

4. (15%) Given below are five observations collected in a regression study on two variables,  $x$  (independent variable) and  $y$  (dependent variable).

$x$	$y$
2	4
3	4
4	3
5	2
6	1

$$\bar{x} = 4, \bar{y} = 2.8, S_{xx} = 10, S_{yy} = 6.8, S_{xy} = -8$$

- Develop the least squares estimated regression equation.
- At 95% confidence, perform a  $t$  test and determine whether or not the slope is significantly different from zero.
- Perform an  $F$  test to determine whether or not the model is significant. Let  $\alpha = 0.05$ .
- Compute the coefficient of determination. Explain it.
- Construct a 95% confidence interval and prediction interval of  $y$  with  $x=5$ .

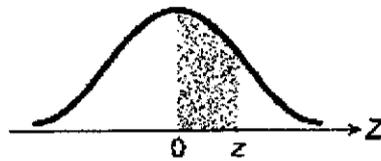
# 國立臺北大學 109 學年度碩士班一般入學考試試題

系(所)組別：統計學系  
 科 目：統計學

第3頁 共5頁  
可 不可使用計算機

統計表

Areas of the standard normal distribution. The entries in this table are the probabilities that a standard normal random variable is between 0 and  $z$  (the shaded area).



Second Decimal Place in  $z$

$z$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4755	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998									
4.0	0.49997									
4.5	0.499997									
5.0	0.4999997									

試題隨卷繳交

接背面

# 國立臺北大學 109 學年度碩士班一般入學考試試題

系(所)組別：統計學系

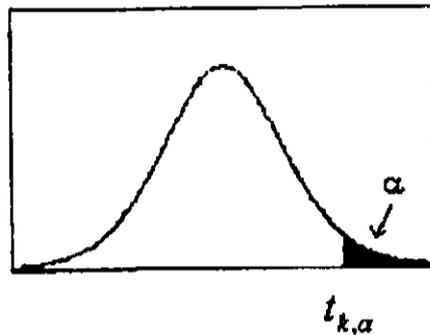
科目：統計學

第4頁 共5頁

可 不可使用計算機

表3.  $t$ 分配表

$$P(t_k \geq t_{k,\alpha}) = \alpha$$



自由度	單尾顯著水準						
	0.1	0.05	0.025	0.01	0.005	0.0025	0.001
1	3.0777	6.3138	12.7062	31.8205	63.6567	127.3213	318.3088
2	1.8856	2.9200	4.3027	6.9646	9.9248	14.0890	22.3271
3	1.6377	2.3534	3.1824	4.5407	5.8409	7.4533	10.2145
4	1.5332	2.1318	2.7764	3.7469	4.6041	5.5976	7.1732
5	1.4759	2.0150	2.5706	3.3649	4.0321	4.7733	5.8934
6	1.4398	1.9432	2.4469	3.1427	3.7074	4.3168	5.2076
7	1.4149	1.8946	2.3646	2.9980	3.4995	4.0293	4.7853
8	1.3968	1.8595	2.3060	2.8965	3.3554	3.8325	4.5008
9	1.3830	1.8331	2.2622	2.8214	3.2498	3.6897	4.2968
10	1.3722	1.8125	2.2281	2.7638	3.1693	3.5814	4.1437
11	1.3634	1.7959	2.2010	2.7181	3.1058	3.4966	4.0247
12	1.3562	1.7823	2.1788	2.6810	3.0545	3.4284	3.9296
13	1.3502	1.7709	2.1604	2.6503	3.0123	3.3725	3.8520
14	1.3450	1.7613	2.1448	2.6245	2.9768	3.3257	3.7874
15	1.3406	1.7531	2.1314	2.6025	2.9467	3.2860	3.7328
16	1.3368	1.7459	2.1199	2.5835	2.9208	3.2520	3.6862
17	1.3334	1.7396	2.1098	2.5669	2.8982	3.2224	3.6458
18	1.3304	1.7341	2.1009	2.5524	2.8784	3.1966	3.6105
19	1.3277	1.7291	2.0930	2.5395	2.8609	3.1737	3.5794
20	1.3253	1.7247	2.0860	2.5280	2.8453	3.1534	3.5518
21	1.3232	1.7207	2.0796	2.5176	2.8314	3.1352	3.5272
22	1.3212	1.7171	2.0739	2.5083	2.8188	3.1188	3.5050
23	1.3195	1.7139	2.0687	2.4999	2.8073	3.1040	3.4850
24	1.3178	1.7109	2.0639	2.4922	2.7969	3.0905	3.4668
25	1.3163	1.7081	2.0595	2.4851	2.7874	3.0782	3.4502
26	1.3150	1.7056	2.0555	2.4786	2.7787	3.0669	3.4350
27	1.3137	1.7033	2.0518	2.4727	2.7707	3.0565	3.4210
28	1.3125	1.7011	2.0484	2.4671	2.7633	3.0469	3.4082
29	1.3114	1.6991	2.0452	2.4620	2.7564	3.0380	3.3962
30	1.3104	1.6973	2.0423	2.4573	2.7500	3.0298	3.3852
35	1.3062	1.6896	2.0301	2.4377	2.7238	2.9960	3.3400
40	1.3031	1.6839	2.0211	2.4233	2.7045	2.9712	3.3069
45	1.3006	1.6794	2.0141	2.4121	2.6896	2.9521	3.2815
50	1.2987	1.6759	2.0086	2.4033	2.6778	2.9370	3.2614
60	1.2958	1.6706	2.0003	2.3901	2.6603	2.9146	3.2317
70	1.2938	1.6669	1.9944	2.3808	2.6479	2.8987	3.2108
80	1.2922	1.6641	1.9901	2.3739	2.6387	2.8870	3.1953
90	1.2910	1.6620	1.9867	2.3685	2.6316	2.8779	3.1833
100	1.2901	1.6602	1.9840	2.3642	2.6259	2.8707	3.1737
200	1.2858	1.6525	1.9719	2.3451	2.6006	2.8385	3.1315
300	1.2844	1.6499	1.9679	2.3388	2.5923	2.8279	3.1176
400	1.2837	1.6487	1.9659	2.3357	2.5882	2.8227	3.1107
500	1.2832	1.6479	1.9647	2.3338	2.5857	2.8195	3.1066
600	1.2830	1.6474	1.9639	2.3326	2.5840	2.8175	3.1039
700	1.2828	1.6470	1.9634	2.3317	2.5829	2.8160	3.1019
800	1.2826	1.6468	1.9629	2.3310	2.5820	2.8148	3.1005
900	1.2825	1.6465	1.9626	2.3305	2.5813	2.8140	3.0993
1000	1.2824	1.6464	1.9623	2.3301	2.5808	2.8133	3.0984

試題隨卷繳交

接下頁

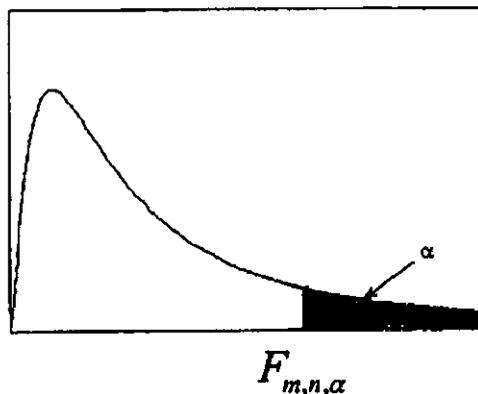
# 國立臺北大學 109 學年度碩士班一般入學考試試題

系(所)組別：統計學系  
 科 目：統計學

第5頁 共5頁  
可 不可使用計算機

表4. F分配表(續3)

(ii)  $\alpha = 0.05$   
 $P(F_{m,n} \geq F_{m,n,\alpha}) = \alpha$



分母自由度 n

		分子自由度 m								
		1	2	3	4	5	6	7	8	9
1	161.448	199.500	215.707	224.583	230.162	233.986	236.768	238.883	240.543	
2	18.5128	19.0000	19.1643	19.2468	19.2964	19.3295	19.3532	19.3710	19.3848	
3	10.1280	9.5521	9.2766	9.1172	9.0135	8.9406	8.8867	8.8452	8.8123	
4	7.7086	6.9443	6.5914	6.3882	6.2561	6.1631	6.0942	6.0410	5.9988	
5	6.6079	5.7861	5.4095	5.1922	5.0503	4.9503	4.8759	4.8183	4.7725	
6	5.9874	5.1433	4.7571	4.5337	4.3874	4.2839	4.2067	4.1468	4.0990	
7	5.5914	4.7374	4.3468	4.1203	3.9715	3.8660	3.7870	3.7257	3.6767	
8	5.3177	4.4590	4.0662	3.8379	3.6875	3.5806	3.5005	3.4381	3.3881	
9	5.1174	4.2565	3.8625	3.6331	3.4817	3.3738	3.2927	3.2296	3.1789	
10	4.9646	4.1028	3.7083	3.4780	3.3258	3.2172	3.1355	3.0717	3.0204	
11	4.8443	3.9823	3.5874	3.3567	3.2039	3.0946	3.0123	2.9480	2.8962	
12	4.7472	3.8853	3.4903	3.2592	3.1059	2.9961	2.9134	2.8486	2.7964	
13	4.6672	3.8056	3.4105	3.1791	3.0254	2.9153	2.8321	2.7669	2.7144	
14	4.6001	3.7389	3.3439	3.1122	2.9582	2.8477	2.7642	2.6987	2.6458	
15	4.5431	3.6823	3.2874	3.0556	2.9013	2.7905	2.7066	2.6408	2.5876	
16	4.4940	3.6337	3.2389	3.0069	2.8524	2.7413	2.6572	2.5911	2.5377	
17	4.4513	3.5915	3.1968	2.9647	2.8100	2.6987	2.6143	2.5480	2.4943	
18	4.4139	3.5546	3.1599	2.9277	2.7729	2.6613	2.5767	2.5102	2.4563	
19	4.3807	3.5219	3.1274	2.8951	2.7401	2.6283	2.5435	2.4768	2.4227	
20	4.3512	3.4928	3.0984	2.8661	2.7109	2.5990	2.5140	2.4471	2.3928	
21	4.3248	3.4668	3.0725	2.8401	2.6848	2.5727	2.4876	2.4205	2.3660	
22	4.3009	3.4434	3.0491	2.8167	2.6613	2.5491	2.4638	2.3965	2.3419	
23	4.2793	3.4221	3.0280	2.7955	2.6400	2.5277	2.4422	2.3748	2.3201	
24	4.2597	3.4028	3.0088	2.7763	2.6207	2.5082	2.4226	2.3551	2.3002	
25	4.2417	3.3852	2.9912	2.7587	2.6030	2.4904	2.4047	2.3371	2.2821	
26	4.2252	3.3690	2.9752	2.7426	2.5868	2.4741	2.3883	2.3205	2.2655	
27	4.2100	3.3541	2.9604	2.7278	2.5719	2.4591	2.3732	2.3053	2.2501	
28	4.1960	3.3404	2.9467	2.7141	2.5581	2.4453	2.3593	2.2913	2.2360	
29	4.1830	3.3277	2.9340	2.7014	2.5454	2.4324	2.3463	2.2783	2.2229	
30	4.1709	3.3158	2.9223	2.6896	2.5336	2.4205	2.3343	2.2662	2.2107	
35	4.1213	3.2674	2.8742	2.6415	2.4851	2.3718	2.2852	2.2167	2.1608	
40	4.0847	3.2317	2.8387	2.6060	2.4495	2.3359	2.2490	2.1802	2.1240	
45	4.0566	3.2043	2.8115	2.5787	2.4221	2.3083	2.2212	2.1521	2.0958	
50	4.0343	3.1826	2.7900	2.5572	2.4004	2.2864	2.1992	2.1299	2.0734	
60	4.0012	3.1504	2.7581	2.5252	2.3683	2.2541	2.1665	2.0970	2.0401	
70	3.9778	3.1277	2.7355	2.5027	2.3456	2.2312	2.1435	2.0737	2.0166	
80	3.9604	3.1108	2.7188	2.4859	2.3287	2.2142	2.1263	2.0564	1.9991	
90	3.9469	3.0977	2.7058	2.4729	2.3157	2.2011	2.1131	2.0430	1.9856	
100	3.9361	3.0873	2.6955	2.4626	2.3053	2.1906	2.1025	2.0323	1.9748	
120	3.9201	3.0718	2.6802	2.4472	2.2899	2.1750	2.0868	2.0164	1.9588	

試題隨卷繳交